Refining security protocols

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Abstract

We propose a development method for security protocols based on stepwise refinement. Our refinement strategy transforms abstract security goals into protocols that are secure when operating over an insecure channel controlled by a Dolev-Yao-style intruder. As intermediate levels of abstraction, we employ messageless guard protocols and channel protocols communicating over channels with security properties. These abstractions provide insights on why protocols are secure and foster the development of families of protocols sharing common structure and properties. We have implemented our method in Isabelle/HOL and used it to develop different entity authentication and key establishment protocols, including realistic features such as key confirmation, replay caches, and encrypted tickets. Our development highlights that guard protocols and channel protocols provide fundamental abstractions for bridging the gap between security properties and standard protocol descriptions based on cryptographic messages. It also shows that our refinement approach scales to protocols of nontrivial size and complexity.

1 Introduction

The fact that the development of even simple security protocols is error prone motivates the use of formal methods to ensure their security. The past decades have witnessed significant progress in post-hoc verification methods for security protocols based on model checking and theorem proving, such as [59, 19, 6, 29, 62]. However, methods for developing security protocols lag behind and protocol design remains more an art than a science.

In our view, a development method should be systematic and hierarchical. By this we mean that the development is decomposed into smaller steps that are easy to understand and that these steps should span well-defined abstraction levels leading the developer from requirements down to cryptographic protocols. Moreover, the resulting protocols should be secure in well-established attacker models and such claims should ideally be supported by machine-checked formal proofs. Stepwise refinement provides such a hierarchical development method. However, most existing refinement-based approaches to developing security protocols [24, 17, 43, 15, 26, 30] fall short in at least one of these desiderata.

In this paper, we advocate a development method based on stepwise refinement that satisfies all requirements in our wish list. Its central element is a four-level refinement strategy,

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Table 1: Levels of the refinement strategy

Level	Name	Features
L0	Security properties	Global, protocol-independent
L1	Guard protocols	Roles, local store, no messages, passive intruder
L2	Channel protocols	Channels with security properties, intruder
L3	Crypto protocols	Cryptographic messages, Dolev-Yao intruder

introduced in Section 3 and summarized in Table 1. This strategy allows developers to build models that incrementally incorporate the system requirements and environment assumptions. Each model constitutes an idealized functionality for subsequent refinements. Safety properties, once proved for a model, are preserved by further refinements.

Level 0 of our refinement strategy consists of abstract, protocol-independent models of *security properties* such as (reachability-based) secrecy and authentication. At Level 1, we introduce *guard protocols* consisting of protocol roles, their local state, and the basic sequencing of the protocol steps. The protocol agents communicate by accessing each other's local states. At Level 2, the agents exchange plaintext messages over communication channels with intrinsic security properties, e.g., confidential or authentic channels. Accordingly, we call these protocols *channel protocols*. Finally, at Level 3, we arrive at standard *cryptographic protocols*, where we replace these messages by cryptographic messages transmitted over insecure channels controlled by a Dolev-Yao intruder. In our method, the functional and security requirements are established at the first two levels, while the last two levels incorporate the environment assumptions, i.e., the hostile distributed environment.

A central and novel feature of our approach is the use of guard protocols (L1) as an intermediate abstraction linking security properties (L0) and message-based protocols (L2–3). Guard protocols enable the straightforward abstract realization of security goals by adding *security guards* as necessary conditions for the execution of certain protocol steps. Different kinds of security guards ensure the preservation of properties such as secrecy, authentication, and recentness. For example, key secrecy means that only authorized agents may know a key. Accordingly, the steps of guard protocols where an agent A learns a key K contain a guard requiring that A is authorized to know K. For authentication, there are guards ensuring that the local state of an agent (partially) agrees with the state of another agent.

The security guards for secrecy and authentication specify conditions on the global state in terms of other agent's local states. This constitutes an abstract form of communication. This abstraction simplifies proofs, but it is not directly implementable in a distributed setting. Hence, we implement these guards at Level 2 by receiving messages on channels with intrinsic security properties. The associated refinement proof naturally gives rise to invariants stating that the receiving of channel messages implies the security guards they implement. These invariants precisely state the security properties guaranteed by the messages. For example, a message containing a key K received on a confidential channel to an agent A may implement a guard authorizing A to learn K. The corresponding invariant guarantees that A is authorized to learn K from this message.

To validate the effectiveness of our refinement strategy, we developed different authentication and key establishment protocols from abstract specifications. In Section 3, we develop two simple unilateral authentication protocols as running examples illustrating our approach: the ISO/IEC 9798-3 protocol and the first two steps of the Needham-Schroeder-Lowe protocol. In

Section 4, we show how to systematically develop an entire family of key transport protocols. This family consists of the Needham-Schroeder Shared-key protocol, the core of Kerberos 4 and Kerberos 5, and the Denning-Sacco protocol. Compared to the running examples from Section 3, these protocols are significantly more complex in their size and message structure and they exhibit additional features and security properties such as the use of timestamps, replay protection caches, encrypted tickets (double encryption), dynamically created communication channels, key confirmation, key freshness, key recentness, and session key compromise.

Contributions We see our work making five main contributions to the state-of-the-art. Our first contribution is methodological and consists of a new method for developing security protocols that are correct-by-construction. Our initial models at Level 0 specify the security goals of the guard protocols at Level 1. These in turn determine the basic structure of entire families of protocols. Using the refinement strategy outlined above, we *systematically* refine these abstract models in a series of well-defined steps to protocols using cryptographic messages sent over an insecure channel. Our refinement strategy aims at proving security properties at the highest possible abstraction level. General results guarantee that these properties are preserved by subsequent refinements.

Our refinement strategy naturally gives rise to straightforward parametrized simulation relations between the state spaces of models at different levels. These relations are instantiated and used in refinement proofs. Moreover, the process of proving refinements helps us discover invariants. For example, the simulation relation linking Levels 2 and 3 usually expresses that the local states of the roles is untouched by the refinement and maps the cryptographic messages at Level 3 to messages on abstract channels at Level 2. An invariant that appears in such refinement proofs states that the honest agents' long-term keys remain secret. This is the natural level of abstraction for this invariant. Typically, the other relevant security properties are already proved in earlier refinements.

Second, our development provides evidence that guard protocols and channel protocols constitute two fundamental abstractions that bridge the gap between security properties and standard protocol descriptions based on cryptographic messages. For the families of authentication and the key establishment protocols we present, the Levels 2 and 3 models refine a common Level 1 ancestor, even though they use different channel types, cryptographic primitives, and communication patterns. Hence, guard protocols enable a unified view of different protocol classes, which disappears at the lower abstraction levels. Moreover, the guarantees about protocol messages we achieve at Level 2 given by the invariants mentioned above are generally absent, or at best informally stated and reasoned about, in other approaches. By formalizing them, our approach fosters clear protocol designs and abstract, simple security proofs. Moreover, in standard protocol descriptions, secrecy and authentication are often not clearly separated (e.g., when using a secure channel providing both properties) or are interdependent (e.g., due to a layered use of cryptographic keys and operations). This is a source of complexity and errors and makes security protocols hard to design and understand. In contrast, guard protocols realize secrecy and authentication properties abstractly and independently. This facilitates the formal development of security protocols and underscores the central role that guard protocols can play in property-driven design approaches.

At the next level, channel protocols allow us to reason about a protocol's security properties at a lower degree of complexity than with the Dolev-Yao intruder. The channels also enable a range of different realizations. For example, we may implement an authentic channel using signatures or MACs. Moreover, the communication structure may change from Level 2 to 3. For instance, an abstract key server may independently distribute keys on secure channels to the initiator and the responder, whereas in the concrete realization the distribution is sequential: one role receives two encrypted copies of the key and forwards one copy to the other role (Section 4). The abstract view represents the essential structure of server-based key distribution. The forwarding is just an implementation detail.

Third, we show how refinement can be used to develop protocols that are secure under a Dolev-Yao intruder model (at Level 3). In contrast, in related work such as [17, 43, 24, 15], the authors do not continue the refinements down to the level of a standard Dolev-Yao model based on an algebra of cryptographic messages; some use ad-hoc, protocol-dependent intruder definitions. This makes it difficult to compare their models with existing work on protocol modeling and verification and to know for which adversaries their properties hold.

Fourth, we show how to develop an entire family of key transport protocols from requirements down to protocols that are secure against a Dolev-Yao intruder. We model realistic features that are often abstracted away, such as replay prevention caches for timestamped messages that achieve strong properties like injective authentication. We have formalized all models and proofs in this paper in the Isabelle/HOL theorem prover. Our formalization includes an general, reusable infrastructure with Isabelle/HOL theories for protocol runs, fresh values, and channels with security properties. Our complete development including the infrastructure theories is available online [65]. Our development shows that our method scales to protocols of realistic complexity. The protocols in our development share both structure and properties as the graph of refinements of our development indicates (see Figure 5). Property preservation through refinements avoids proof duplication and fosters well-structured proofs.

Our final contribution is a comprehensive definitional extension of Isabelle/HOL with a theory of refinement that is based on simulation and borrows elements from [4, 2]. We define an implementation relation on models including a notion of observation, derive proof rules for invariants and refinement, and show that refinement is a sound method for establishing the implementation relation between models.

This article is based on [63, 64]. The main difference is that we have extended our method with a stronger attacker who is able to compromise secrets. As a consequence, we obtain stronger security guarantees. This required modifications throughout the refinement tree. At the top-level, we added new events modeling the leakage of messages. At the lower levels, this required the modification of guards, the generalization of simulation relations, as well as the adaptation of several invariants. Moreover, we were able to shift several invariants from L3 to L2 and reduce their overall number, which further highlights the use of our approach to prove properties at the highest possible level of abstraction.

Organization The remainder of this paper is organized as follows. In Section 2, we introduce Isabelle/HOL, notational conventions, and summarize the theory of refinement that we have embedded in Isabelle/HOL. In Section 3, we present our four-level refinement strategy for security protocols and illustrate its application by deriving two simple authentication protocols. In Section 4, we report on our development of a family of key transport protocols. In Section 5, we further discuss related work and we draw conclusions in Section 6.

2 Preliminaries

2.1 Isabelle/HOL and notation

Isabelle is a generic, tactic-based theorem prover. We have used Isabelle's implementation of higher-order logic, Isabelle/HOL [58], for our developments. HOL can roughly be seen as logic on top of a functional programming language. We assume that the reader has basic familiarity with both logic and typed functional programming. Proof automation in Isabelle is supported by a rewrite-based simplifier and a tableau procedure. These are invoked in isolation or combined using proof tactics.

To enhance readability, we use standard mathematical notation where possible and blur the distinction between types and sets. We also drop typing information unless it is essential to understand a definition.

We use two definitional equalities: $\equiv$ for terms and $\triangleq$ for types. We define partial functions by $A \rightharpoonup B \triangleq A \to B_\perp$, where $B_\perp \triangleq \mathsf{Some}(B) \mid \perp$; note that we will often elide single-argument constructors, e.g., writing $\mathsf{Some}(x)$ simply as x. The term $f(x \mapsto y)$ denotes the function that behaves like f, except that it maps x to y. For a function or binary relation $R \subseteq A \times B$ and a set $X \subseteq A$, we define the image of X under R by $R(X) \equiv \{y \in B \mid \exists x \in X.\, (x, y) \in R\}$. The inductive type of lists is defined by $\mathit{list}(A) \triangleq \mathsf{Nil} \mid \mathsf{Cons}(A, \mathit{list}(A))$, where Nil is the empty list and $\mathsf{Cons}(a, l)$ is the list built by prefixing the element $a \in A$ to the list $l \in \mathit{list}(A)$. We usually write Nil as $[]$ and $\mathsf{Cons}(a, l)$ infix as $a\#l$. We also write $[1, 2, 3]$ for $1\#2\#3\#[]$. We define multisets over A by $\mathit{multiset}(A) \triangleq A \to \mathbb{N}$. For multisets $m, n \in \mathit{multiset}(A)$, the term $m(e)$ indicates the multiplicity of e in m and union is defined by $(m \uplus n)(e) = m(e) + n(e)$. Record types may be defined, such as $\mathit{point} \triangleq (\!|\, x \in \mathbb{N}, y \in \mathbb{N} \,|\!)$ with elements like $r = (\!|\, x = 1, y = 2 \,|\!)$ and projections $r.x$ and $r.y$. The term $r(\!|\, x := 3 \,|\!)$ denotes r, where x is updated to 3, i.e., $(\!|\, x = 3, y = 2 \,|\!)$. The type $\mathit{cpoint} \triangleq \mathit{point} + (\!|\, c \in \mathit{color} \,|\!)$ extends point with a field of type color. For record types T and U including fields F, we define the field identity relation $\Pi_F^{T,U} \equiv \{(r, s) \in T \times U \mid \bigwedge_{x \in F} r.x = s.x\}$. If U has exactly the fields F, the field projection function $\pi_F^{T,U} \colon T \to U$ projects T to U. We will drop the superscripts T and U from $\Pi_F^{T,U}$ and $\pi_F^{T,U}$ when they are clear from the context.

2.2 Refinement theory

A development by refinement starts from a set of system requirements and environment assumptions. We then construct a series of models resulting in a system that fulfills the requirements provided it runs in an environment satisfying the assumptions. We summarize the refinement theory that we developed in Isabelle/HOL. It is inspired by [4, 2].

2.2.1 Specification and implementation

We define the structure of our models and we formalize the meaning of implementation.

Definition 2.1. A *specification* is a triple of the form $S = (T, O, \mathit{obs})$, where $T = (\Sigma, \Sigma_0, \to)$ is a *transition system* with state space Σ, set of initial states $\Sigma_0 \subseteq \Sigma$, and transition relation $\to \subseteq \Sigma \times \Sigma$. The *observation function* $\mathit{obs} : \Sigma \to O$ maps states to elements of a set of observations O.

The definition of transition systems is standard. The observation function *obs* specifies which state information is visible to an outside observer. This function can be freely chosen and is typically just a projection on a subset of the state variables. We will use the term *model* as a synonym for specification.

We will work with structured specifications, where the state space Σ is a record type, i.e., a set of tuples of state variables, and the transition relation $\rightarrow$ is a finite union of parametrized relations, called *events*. Events have the form

$$evt(\overline{x}) = \{(s, s') \mid G(\overline{x}, s) \wedge s'.\overline{v} := \overline{f}(\overline{x}, s)\},$$

where $\overline{\cdot}$ denotes vectors. Here, $\overline{x}$ are the event's parameters, $\overline{v}$ are the state variables, $G(\overline{x}, s)$ is a conjunction of *guards*, and $s'.\overline{v} := \overline{f}(\overline{x}, s)$ is an *action* with *update functions* $\overline{f}$. The guards depend on the parameters $\overline{x}$ and the current state s and determine when the event is enabled. The action is syntactic sugar denoting the relation $s' = s(\!|\, \overline{v} := \overline{f}(\overline{x}, s) \,|\!)$, i.e., the simultaneous assignment of values $\overline{f}(\overline{x}, s)$ to the variables $\overline{v}$ in the state s, yielding the state s'. The concrete parameters for an event's execution can be thought of as being chosen non-deterministically by the environment. We assume that all models include the distinguished event *skip*, which denotes the identity relation and thus models stuttering steps.

Example 2.2. Consider an abstract file transfer protocol specification

$$S_f \equiv ((\Sigma_f, \Sigma_f, \rightarrow_f), O_f, id).$$

Here $\Sigma_f \triangleq (\!|\, f, g \in \mathit{file} \,|\!)$ with $\mathit{file} \triangleq I \rightarrow D$, where I is a finite index set and D is a set of data blocks, and $\rightarrow_f \equiv \mathit{xfer}_f$. The event $\mathit{xfer}_f \equiv \{(s, s') \mid s'.g := s.f\}$ transfers the file f in one shot to g. All states are possible initial states and the entire state is observable, i.e., $O_f = \Sigma_f$ and the observation function is the identity.

The set of behaviors of S, $beh(S)$, consists of the finite sequences $s_0 \cdots s_n$ of states that start in an initial state $s_0 \in \Sigma_0$ and are linked by transitions in $\rightarrow$, that is, $s_i \rightarrow s_{i+1}$ for $0 \leq i < n$. The set $reach(S)$ denotes the set of reachable states. Since we only consider safety properties (in fact, invariants), it suffices to model just finite behaviors. The sets of *observable behaviors* and *reachable observations* of the specification S are defined by $obeh(S) \equiv obs(beh(S))$ and $oreach(S) \equiv obs(reach(S))$, where *obs* is applied to behaviors element-wise. Our notion of one specification implementing another is defined by the inclusion of their observable behaviors.

Definition 2.3. We say S_c *implements* S_a *via the mediator function* $\pi : O_c \rightarrow O_a$ if we have $\pi(obeh(S_c)) \subseteq obeh(S_a)$.

The mediator function specifies how to abstract observations of S_c to observations of S_a. It enables details to be added to observations during implementation. We consider two types of invariants: internal invariants are supersets of $reach(S)$ and external invariants are supersets of $oreach(S) \equiv obs(reach(S))$. We use internal invariants to strengthen simulation relations in refinement proofs (see Example 2.9 below). The observation function determines which properties can be expressed as external invariants. These invariants are important since they are preserved by implementations.

Proposition 2.4 (**Invariant preservation**). *Suppose* S_c implements S_a via the mediator function π *and* $oreach(S_a) \subseteq J$ *for some* $J \subseteq O_a$. *Then* $\pi(oreach(S_c)) \subseteq J$.

Proposition 2.4 guarantees a well-defined notion of property preservation for a series of implementations. We use refinement as a proof method to establish implementations.

$$\begin{tikzcd} s \arrow[r, dashed, "evt_a"] \arrow[d, "R"'] & s' \arrow[d, dashed, "R"] \\ t \arrow[r, "evt_c"] & t' \end{tikzcd} \qquad \begin{tikzcd} s \arrow[r, "obs_a"] \arrow[d, "R"] & o \\ t \arrow[r, "obs_c"] & p \arrow[u, "\pi"'] \end{tikzcd}$$

Figure 1: Refinement of events (left) and observations (right).

2.2.2 Refinement

Our notion of refinement is based on standard simulation [49], which we extend to account for observations [3]. Figure 1 illustrates the second and third point of the following definition.

Definition 2.5. We say $S_c = ((\Sigma_c, \Sigma_{c,0}, \rightarrow_c), O_c, obs_c)$ *refines* $S_a = ((\Sigma_a, \Sigma_{a,0}, \rightarrow_a), O_a, obs_a)$ using the simulation relation $R \subseteq \Sigma_a \times \Sigma_c$ and the *mediator function* $\pi : O_c \rightarrow O_a$, written $S_c \sqsubseteq_{R,\pi} S_a$, if the following three conditions hold.

1. Each concrete initial state is related to some abstract initial state, i.e.,

$$\Sigma_{c,0} \subseteq R(\Sigma_{a,0}).$$

2. For each concrete event $evt_c(\overline{x})$, there is an abstract event $evt_a(\overline{z})$ and *witness functions* $\overline{w}$, mapping concrete parameters $\overline{x}$ to abstract parameters $\overline{w}(\overline{x})$ such that $evt_a(\overline{w}(\overline{x}))$ simulates $evt_c(\overline{x})$, i.e.,

$$R; evt_c(\overline{x}) \subseteq evt_a(\overline{w}(\overline{x})); R,$$

where ';' is forward relational composition. The plain and dashed arrows in Figure 1 (left) correspond to the given part on the left-hand side and the sought part on the right-hand side of the inclusion above.

3. *R respects observations mediated by π*, i.e., for all $(s, t) \in R$,

$$obs_a(s) = \pi(obs_c(t)).$$

We say S_c *refines* S_a *using* π, written $S_c \sqsubseteq_\pi S_a$, if $S_c \sqsubseteq_{R,\pi} S_a$ for some R.

In our method, new concrete events usually refine the abstract *skip* event. The intuition is that these events correspond to unobservable stuttering steps in the abstract model. Note that, if R^{-1} is functional, condition (3) boils down to the equation $R^{-1}; obs_a = obs_c; \pi$. Choosing $obs_a = obs_c = id$ and $\pi = R^{-1}$ yields maximal observations. Otherwise, if R^{-1} is non-functional, there is a tension between what can be made observable and the fact that the mediators are functional.

Let the concrete and abstract system have state variables $\overline{v}$ and $\overline{u}$, respectively. Moreover, suppose the event $evt_c(\overline{x})$ has guards G_c and update functions $\overline{f_c}$ and the abstract event $evt_a(\overline{z})$ has guards G_a and update functions $\overline{f_a}$. Condition 2 can be decomposed into two proof obligations, called *guard strengthening* and *action refinement*, both under the premises $(s, t) \in R$ and $G_c(\overline{x}, t)$.

(GRD) $G_a(\overline{w}(\overline{x}), s)$.

(ACT) $(s(\!|\, \overline{u} := \overline{f_a}(\overline{w}(\overline{x}), s) \,|\!), t(\!|\, \overline{v} := \overline{f_c}(\overline{x}, t) \,|\!)) \in R$.

Guard strengthening requires that if the concrete event is enabled then so is the abstract one. Action refinement expresses that the two states resulting from the execution of the abstract and concrete actions are again related by R.

Refinement is reflexive and transitive. Moreover, refinement is a sound (but incomplete [2, 44]) method to establish implementation.

Proposition 2.6 (Pre-order). *We have (i) $S \sqsubseteq_{id} S$ and (ii) $S_3 \sqsubseteq_\rho S_2$ and $S_2 \sqsubseteq_\pi S_1$ imply $S_3 \sqsubseteq_{\pi \circ \rho} S_1$.*

Proposition 2.7 (Soundness of refinement). *If $S_c \sqsubseteq_\pi S_a$ then S_c implements S_a via the mediator function π.*

The combination of Propositions 2.4, 2.6, and 2.7 ensures that requirements and assumptions, once established as external invariants, are preserved by subsequent refinements. Note that this does not generally hold for internal invariants. For later reference we state the following corollary.

Corollary 2.8 (Invariant preservation II). *Suppose $S_c \sqsubseteq_\pi S_a$ and $oreach(S_a) \subseteq J$ for some $J \subseteq O_a$, where O_a is S_a's set of observations. Then $\pi(oreach(S_c)) \subseteq J$.*

Example 2.9. We define a "protocol" implementing the file transfer specification S_f by $S_p \equiv (T_p, O_p, obs_p)$, where $T_p = (\Sigma_p, \Sigma_{p,0}, \rightarrow_p)$ and $\Sigma_p \triangleq \Sigma_f + (\!|\, b \in I \rightharpoonup D \,|\!)$ extends the state Σ_f with a buffer b. The set $\Sigma_{p,0}$ consists of initial states of the form $(\!|\, f = f_0, g = g_0, b = \emptyset \,|\!)$ for some $f_0, g_0 \in \mathit{file}$ and the empty buffer b. The protocol non-deterministically transfers blocks of the file f into the buffer b from where it is assigned to g, once the transfer is completed. The transition relation $\rightarrow_p \equiv xfer_p \cup \bigcup_{i \in I} blk_p(i)$ is the union of two events:

$$blk_p(i) \equiv \{(t, t') \mid i \in I \setminus dom(t.b) \wedge t'.b := t.b(i \mapsto f(i))\}$$

and $xfer_p \equiv \{(t, t') \mid dom(b) = I \wedge t'.g := t.b\}$. The observation function $obs_p \equiv \pi_f$ projects the state Σ_p to the observations $O_p = \Sigma_f$.

Let us try to establish a refinement between S_p and S_f, using the simulation relation $R \equiv \Pi_f \subseteq S_f \times S_p$, i.e., the inverse of the projection $\pi_f : \Sigma_p \to \Sigma_f$, and the identity mediator function $\pi \equiv id$. We focus on point (2), where we must show that $blk_p(i)$ refines *skip* and that $xfer_p$ refines $xfer_f$. The guard strengthening **(GRD)** proof obligation is trivial in both cases, since the abstract guards are true. The action refinement **(ACT)** proof obligation for $blk_p(i)$ and *skip* (the identity relation) requires showing $(s, t') \in \Pi_f$ for $t' = t(\!|\, b := t.b(i \mapsto f(i)) \,|\!)$, assuming $(s, t) \in \Pi_f$ and $i \in I$. This holds trivially, since $t'.g = t.g = s.g$. In the action refinement for $xfer_p$ and $xfer_f$, we must show that $(s(\!|\, g := s.f \,|\!), t(\!|\, g := t.b \,|\!)) \in \Pi_f$ assuming $(s, t) \in \Pi_f$ and $dom(b) = I$. To prove this, we need additional information about the relation between b and f, which we express as the internal invariant $I_p \equiv \{t \in \Sigma_p \mid \forall i \in dom(b).\ t.b(i) = t.f(i)\}$ of S_p. We establish this invariant separately and use it to strengthen the simulation relation to $R \equiv \Pi_f \cap (\Sigma_f \times I_p)$.

In further refinements, one could develop a more realistic implementation, for example, by eliminating non-determinism and by modeling a communication medium such as an unreliable channel with acknowledgement messages.

3 Security protocol refinement

We now present our framework for security protocol development by refinement. Each development starts by defining the system requirements and the environment assumptions. The environment assumptions include the attacker model and the cryptographic setup. Given these elements we need a *refinement strategy* telling us in which order to incorporate them into our models. The crucial point is that requirements and assumptions, once modeled, are *preserved* by subsequent refinement steps (Corollary 2.8).

The following four-level refinement strategy guides our developments, where each level may itself consist of several refinement steps.

Level 0 *Security properties.* We give abstract, protocol-independent specifications of secrecy and authentication properties. The models' states contain just enough structure to formulate these properties as invariants and define a few abstract events satisfying these invariants.

Level 1 *Guard protocols.* These are abstract protocols without message passing. We introduce protocol roles, local states of agents, and basic protocol steps. Agents read data directly from other agents' local states, whereby guards enforce the data's security properties.

Level 2 *Channel protocols* communicate over abstract channels with security properties, such as confidential and authentic channels. The intruder may eavesdrop messages on non-confidential channels and fake messages on non-authentic channels. No cryptography is used.

Level 3 *Cryptographic protocols.* The messages on the abstract channels from Level 2 are now implemented using cryptographic messages sent over insecure channels. A standard Dolev-Yao intruder completely controls the network.

In the rest of this section, we introduce protocol-independent infrastructure for modeling and reasoning about security properties, protocol runs, fresh values, channels with security properties, and intruder behavior. We also describe for each level the relevant structures such as type definitions, state variables, and simulation relations. As running examples, we develop two simple authentication protocols. In Section 4, we will apply this framework to a more complex development and construct a family of key establishment protocols.

Several explanations are in order. First, while we formulate secrecy and authentication as Level 0 models and establish them by refinement, we express and prove other security properties such as key freshness as invariants. Second, in our setup, agents, keys, nonces, and timestamps have different types and we use variables of these types in our protocol models. However, other options are available. We can easily formalize protocols in an untyped fashion in our framework by using only message variables, which represent arbitrary messages. Finally, the guarantees we obtain in our framework hold for an unbounded number of protocol instances.

3.1 Entity authentication protocols

To illustrate our methodology, we develop two unilateral authentication protocols. Both protocols are based on a standard challenge-response pattern, where the initiator sends a nonce as a challenge to the responder who returns it in a cryptographically transformed form that

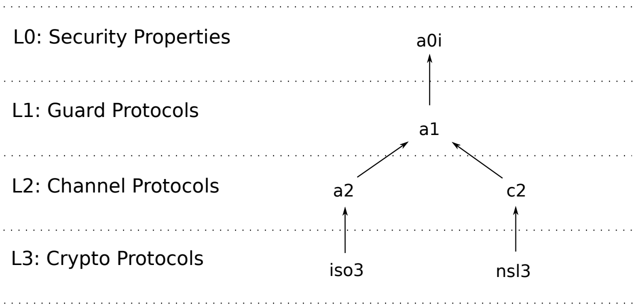


Figure 2: Refinement graph for authentication protocols.

authenticates him to the initiator. The first protocol is the signature-based two-pass ISO/IEC 9798-3 standard [36]. The second, which we call NSL/2, consists of the first two steps of the Needham-Schroeder-Lowe protocol [41] and uses public-key encryption.

We start by specifying the system requirements and by making our assumptions about the environment explicit. Our notion of entity authentication is based on the strong property of injective agreement [42]. A protocol satisfies this property, if given two roles r and r' and the data d then for each protocol thread executing role r there exists a unique thread executing role r' with whom it agrees on the data d.

Requirement R1 (Protocol roles). The protocol has two roles, which we call initiator and responder.

Requirement R2 (Entity authentication). The initiator injectively agrees with the responder on the initiator's nonce and possibly on additional data.

We assume a standard Dolev-Yao intruder that we identify as usual with the communication network. Moreover, we make two assumptions about agent corruption and the cryptographic setup.

Assumption A1 (Dolev-Yao intruder). The intruder controls the network. He receives all messages sent and he can build and send messages composed from their parts obtained by decomposing received messages using the cryptographic keys he knows.

Assumption A2 (Static corruption of agents). An arbitrary fixed subset of agents is corrupted,

whereby their long-term keys are exposed to the intruder.

Assumption A3 (Cryptographic setup). The requisite cryptographic keys are distributed prior to protocol execution.

Development overview Figure 2 summarizes the refinements in our development in a *refinement graph*. Each node represents a model and each arc $m \to m'$ represents a refinement $m \sqsubseteq_\pi m'$ for some mediator function π, not shown.

We progress from the initial model *a0i* representing the intended injective agreement property at Level 0 (Section 3.3) to the guard protocol *a1* at Level 1 (Section 3.4). This guard protocol is refined into two channel protocols, *a2* and *c2*, based on authentic and confidential channels, respectively (Section 3.5). Finally, we realize these two protocols as cryptographic protocols, *iso3* and *nsl3* (Section 3.6). In the following, we will restrict our presentation to the left path in the refinement graph. We omit the models *c2* and *nsl3*, whose development is similar to *a2*'s and *iso3*'s.

3.2 General setup

We describe our formalization of atomic messages: agents, keys, nonces, and numbers. We start by defining a type of agents, *agent*. We assume a subset *bad* of dishonest agents (with complement *good*) and an honest server $\mathsf{S} \in \mathit{good}$.

We also need a mechanism to generate fresh nonces and keys. We assume a type *rid* of identifiers that we will use to uniquely identify protocol runs at Levels 1–3. From this type, we derive the type of freshness identifiers as $\mathit{fresh} \triangleq \mathsf{mkf}(\mathit{rid}, \mathbb{N})$, which has a single constructor mkf. We write $\mathsf{mkf}(R, i)$ as $R\$i$. In protocol specifications, we often use global constants to distinguish different fresh values of a run R, such as $R\$\mathsf{sk}$, $R\$\mathsf{na}$, or $R\$\mathsf{nb}$. We write both constructors and constants in sans serif font. This setup allows us to derive an arbitrary number of unique freshness identifiers from each protocol run identifier. We define the types of nonces, session keys, and all keys as follows.

$$\mathit{nonce} \triangleq \mathit{fresh} \qquad \mathit{sesk} \triangleq \mathit{fresh} \qquad \mathit{key} \triangleq \mathsf{sesK}(\mathit{sesk}) \mid \mathsf{ltK}(\mathit{ltk}).$$

Nonces and session keys both use freshness identifiers. The type of long-term keys, *ltk*, is left unspecified at this point.

Finally, we define the type *atom* of atomic messages as the disjoint sum of the types of agents, nonces, keys, and numbers:

$$\mathit{atom} \triangleq \mathsf{aAgt}(\mathit{agent}) \mid \mathsf{aNon}(\mathit{nonce}) \mid \mathsf{aKey}(\mathit{key}) \mid \mathsf{aNum}(\mathbb{N}).$$

We use numbers mainly as timestamps. We will usually omit constructors from atomic messages and use a notational convention instead. In particular, we use A, B, C for agents, $N, \mathit{Na}, \mathit{Nb}$ for nonces, K, Kab for session keys, and T, Ta, Ts for timestamps.

Many protocols assume a setup of long-term keys, which is established out-of-band before the protocol starts. We model this by assuming an abstract (uninterpreted) key setup $\mathit{keySetup} \subseteq \mathit{key} \times \mathit{agent}$ defining the initial key knowledge of each agent. The relevant definitions are deferred to Level 3 (Section 3.6), since they are protocol-dependent. For example, the protocol may use a PKI or a shared-key setup. The set of statically corrupted keys is derived from the key setup as the keys initially known by dishonest agents: $\mathit{corrKey} \triangleq \mathit{keySetup}^{-1}(\mathit{bad})$.

3.3 Level 0 – Security properties

We present abstract, protocol-independent models of secrecy and authentication and we formalize and prove their relevant properties as external invariants. Each protocol development starts with the formalization of the protocol's security requirements. This is achieved by appropriately instantiating these Level 0 models. We will later show that our guard protocol models at Level 1 refine these instantiated models, thus establishing the respective requirements (by Corollary 2.8).

3.3.1 Secrecy

Our first model abstractly formalizes secrecy including a notion of dynamic compromise. We introduce three state variables. The first two, *kn* and *az*, are relations between data (of polymorphic type δ) and agents. In particular, $(d, A) \in s.kn$ means that agent A knows data d in state s, and $(d, A) \in s.az$ means that agent A is authorized to know data d in state s. The third state variable, *lk*, records compromised data that has leaked to the intruder. The entire state is observable.

$$\begin{array}{lll}\Sigma_{s0}(\delta) \triangleq (\!| \, kn \in \mathcal{P}(\delta \times agent), & & \texttt{-- knowledge relation} \\ \qquad\qquad az \in \mathcal{P}(\delta \times agent), & & \texttt{-- authorization relation} \\ \qquad\qquad lk \in \mathcal{P}(\delta) \, |\!). & & \texttt{-- leaked data}\end{array}$$

Secrecy can be expressed as the property that all knowledge is either authorized or leaked.

$$secrecy \equiv \{s \mid s.kn \subseteq s.az \cup (s.lk \times agent)\}.$$

In other words, an agent is allowed to know a secret only if he is authorized to do so or if the secret has leaked to the intruder. We allow any state satisfying this property to be an initial state.

Events This model has events for secret generation, for learning secrets, and for leaking them to the intruder. The secret generation event is parametrized by the data d, an agent A, and the intended group of agents G sharing d.

$$\begin{array}{ll}gen_{s0}(d, A, G) \equiv \{(s, s') \mid & \\ \quad \texttt{-- guards} & \\ \quad d \notin dom(s.kn) \wedge & \texttt{-- } d \texttt{ is fresh} \\ \quad A \in G \wedge & \texttt{-- } A \texttt{ is a member of } G \\ \quad \texttt{-- actions} & \\ \quad s'.kn := s.kn \cup \{(d, A)\} \wedge & \\ \quad s'.az := s.az \cup (\{d\} \times (\text{if } G \cap bad = \emptyset \text{ then } G \text{ else } agent))\}. & \end{array}$$

The guards require that d is fresh, i.e., not known to any agent, and that A belongs to the group G. The first action adds the pair (d, A) to the knowledge relation $s.kn$. The second action updates the authorization relation with $\{d\} \times G$ if all agents in G are honest and with $\{d\} \times agent$ otherwise. That is, if the group G contains a dishonest member, there is no point in restricting access to d.

In the secret-learning event, an agent B learns the secret d provided he is authorized or d has leaked.

$$\begin{array}{ll}learn_{s0}(d, B) \equiv \{(s, s') \mid & \\ \quad \texttt{-- guards} & \\ \quad (d, B) \in s.az \cup (s.lk \times agent) \wedge & \texttt{-- } B \texttt{ authorized to know } d \texttt{ or } d \texttt{ leaked} \\ \quad \texttt{-- action} & \\ \quad s'.kn := s.kn \cup \{(d, B)\} \}. & \end{array}$$

From a secrecy perspective, it is irrelevant from whom B learns d. Authentication aspects will be covered separately. The final event of *s0* leaks some data d to the intruder.

$$
\begin{array}{ll}
\mathit{leak}_{s0}(d) \equiv \{(s,s') \mid & \\
\quad \texttt{-- guards} & \\
\quad d \in \mathit{dom}(s.\mathit{kn}) \wedge & \texttt{-- someone knows } d \\
& \\
\quad \texttt{-- action} & \\
\quad s'.\mathit{lk} := s.\mathit{lk} \cup \{d\} \,\}. &
\end{array}
$$

The model *s0* clearly preserves secrecy.

Proposition 3.1. *Let s0 be the above specification. Then* $\mathit{oreach}(s0) \subseteq \mathit{secrecy}$.

3.3.2 Authentication

Our notion of authentication is based on Lowe's agreement [42]. Informally, a role r *non-injectively agrees* with the role r' on the data d if whenever an honest agent A in role r terminates a run, apparently with an honest agent B in role r', then there is a run of agent B in role r' with whom he agrees on the participants, their roles, and the data d. This agreement is *injective* if to each such run of role r corresponds a unique run of role r'.

We formulate two models, *a0n* and *a0i*, that represent a minimal, extensional variant of non-injective and injective agreement using signals, which indicate particular stages of each role's progress (e.g., termination). The state record has as its single field an initially empty multiset of signals, *sigs*. The entire state is observable.

$$
\begin{aligned}
\mathit{signal}(\delta) &\triangleq \mathsf{Running}(\mathit{list}(\mathit{agent}) \times \delta) \mid \mathsf{Commit}(\mathit{list}(\mathit{agent}) \times \delta), \\
\Sigma_{a0}(\delta) &\triangleq (\!|\ \mathit{sigs} \in \mathit{multiset}(\mathit{signal}(\delta))\ |\!).
\end{aligned}
$$

There are two signals: $\mathsf{Running}(h,d)$ and $\mathsf{Commit}(h,d)$, where h is a list of agents and d is data of polymorphic type δ that is instantiated later. The agreement on the data d is, by convention, between the first two agents in h and assumes the honesty of all agents in h. In the simplest case, h includes the two agents participating in the agreement, but in some cases it is necessary to include some other agents in h, for instance agents relaying messages.

Non-injective agreement states that if the agents in h are honest and there is a $\mathsf{Commit}(h,d)$ signal (thought to be raised by the first agent in h), then there is a matching $\mathsf{Running}(h,d)$ signal (raised by the second agent in h).

$$
\begin{aligned}
&\mathit{ni\,agree}_{a0n} \equiv \{s \mid \forall h, d. \\
&\quad h \subseteq \mathit{good} \wedge s.\mathit{sigs}(\mathsf{Commit}(h,d)) > 0 \rightarrow s.\mathit{sigs}(\mathsf{Running}(h,d)) > 0\}.
\end{aligned}
$$

Injective agreement strengthens this by requiring that the number of $\mathsf{Commit}(h,d)$ signals is not greater than the number of matching $\mathsf{Running}(h,d)$ signals.

$$
\mathit{iagree}_{a0i} \equiv \{s \mid \forall h, d.\ h \subseteq \mathit{good} \rightarrow s.\mathit{sigs}(\mathsf{Commit}(h,d)) \leq s.\mathit{sigs}(\mathsf{Running}(h,d))\}.
$$

Events The models *a0n* and *a0i* have two events, $running(h, d)$ and $commit(h, d)$, which add the corresponding signal to the multiset $s.sigs$. The first event adds a $\mathsf{Running}(h, d)$ signal to the multiset $s.sigs$.

$$running_{a0n}(h, d) \equiv \{(s, s') \mid s'.sigs := s.sigs \uplus \{\mathsf{Running}(h, d)\}\,\}.$$

The second event adds a $\mathsf{Commit}(h, d)$ signal to the multiset $s.sigs$. Its guard requires that there is a matching $\mathsf{Running}(h, d)$ signal if the agents in h are honest. This ensures that the invariant $niagree_{a0n}$ is preserved.

$$\begin{array}{l} commit_{a0n}(h, d) \equiv \{(s, s') \mid \\ \quad \texttt{-- guards} \\ \quad h \subseteq good \rightarrow s.sigs(\mathsf{Running}(h, d)) > 0 \wedge \\ \quad \texttt{-- actions} \\ \quad s'.sigs := s.sigs \uplus \{\mathsf{Commit}(h, d)\}\,\}. \end{array}$$

The honesty condition weakens the guard just enough to accommodate the protocol's interaction with an explicit intruder in later refinements.

The model *a0i* for injective agreement is the same except for the guard in the event $commit_{a0i}(h, d)$, which we strengthen as follows to preserve the stronger invariant $iagree_{a0i}$.

$$h \subseteq good \rightarrow s.sigs(\mathsf{Commit}(h, d)) < s.sigs(\mathsf{Running}(h, d)).$$

It is easy to see that *a0i* refines *a0n*. The properties of the models *a0n* and *a0i* are summarized as follows.

Proposition 3.2. *The models a0n and a0i defined above have the following properties: (i)* $oreach(a0n) \subseteq niagree_{a0n}$*, (ii)* $oreach(a0i) \subseteq iagree_{a0i}$*, and (iii)* $a0i \sqsubseteq_{id,id} a0n$*.*

Since the variable *sigs* is observable, these invariants are preserved by further refinements (Corollary 2.8).

Example 3.3 (Formalization of authentication properties). Our example development starts with a formalization of the main security property: entity authentication by injective agreement (**R2**). We do this by instantiating the model *a0i*. Since we would like our entity authentication protocols to achieve injective agreement between the initiator A and the responder B on a nonce Na generated by A and also on a nonce Nb generated by B, we instantiate the type of data δ in the model *a0i* to $nonce \times nonce$.

This model is very abstract. Further refinement steps are needed to obtain a protocol that is executable in the intended distributed environment and is secure against a Dolev-Yao intruder as described by the environment assumptions.

3.3.3 Other types of security property specifications

In contrast to establishing security properties by refinements of L0 models, we may express (and prove) security properties directly at the protocol level (i.e., at Level 1, 2, or 3) in one of two ways.

First, the property may be ensured by a *guard*, whereby the property is established by construction so that no extra proof is required. An example is a guard checking the validity of a timestamp to ensure the recentness of an associated session key. Second, we may formulate and prove the property as an *external invariant* of the protocol. This is how we express the freshness of session keys. An advantage of using the refinement of abstract models over these two alternatives is that the abstract models are protocol-independent. This enables clear and uniform property specifications. In contrast, invariants that are formulated at the protocol level (see, e.g., [59]) must be specified individually and tend to be more complex.

3.4 Level 1 – Guard protocols

We now introduce protocol roles and runs. A run is a thread executed by some agent in a given role. Each run has a local memory holding state information. At this abstract level, runs share information by reading each other's memory. We call such protocols *guard protocols*. Of course, this kind of communication by reading another thread's memory is unrealistic in a distributed setting. Hence, at Level 2, we will refine this abstract form of communication by passing messages over communication channels.

3.4.1 State

Guard protocols have at least one state variable, *runs*, which is a partial function mapping each run identifier (of type *rid*) in its domain to the run's *local store*. This local store consists of the executed role, the participants, and a *frame* recording role-specific information. As we focus here on two-party protocols, we model participants as a pair of agents. Similarly, we fix the roles to an initiator (the first agent), a responder (the second agent), and possibly an additional fixed server S. This could easily be generalized to handle an arbitrary number of roles. The frame is a list of atomic messages that the run acquires during its execution.

$$\begin{aligned}
\mathit{role} &\triangleq \mathsf{Init} \mid \mathsf{Resp} \mid \mathsf{Serv}, \\
\mathit{frame} &\triangleq [\mathit{atom}], \\
\mathit{runsT} &\triangleq \mathit{rid} \rightharpoonup \mathit{role} \times \mathit{agent} \times \mathit{agent} \times \mathit{frame}, \\
\Sigma_1 &\triangleq (\!|\ \mathit{runs} \in \mathit{runsT}\ |\!), \\
\Sigma_1' &\triangleq \Sigma_1 + (\!|\ \mathit{leak} \in \mathcal{P}(\delta \times \gamma)\ |\!).
\end{aligned} \tag{1}$$

Here, we have schematically defined the state of a Level 1 protocol by the record type Σ_1. In concrete models, this state may contain additional variables. In particular, L1 specifications that refine the secrecy model *s0* extend this state with an additional variable $\mathit{leak} \in \mathcal{P}(\delta \times \gamma)$ that refines the variable *lk* of *s0* as indicated in Σ_1'. Here, δ represents the type of the leaked data and γ the type of an (optional) session context recorded together with the leaked data. For example, to model session key compromise and record the intended partners sharing the key, we set $\delta = \mathit{key}$ and $\gamma = \mathit{agent} \times \mathit{agent}$. In Section 4.4, we will present an example using the variable *leak* in detail. We assume that (at least) the variable *runs* is observable. All subsequent refinements keep the variable *runs*, but may add atoms to the run's frames. This allows us to prove external invariants about the protocol runs and inherit them through the refinements.

3.4.2 Events

Each event executes a protocol step of a run by an agent in a particular role. The sequencing of events within a role is determined by *local guards* reading a run's local store. For example, the guard $runs(R) = (\mathsf{Init}, A, B, [Nb])$ expresses that the event executes a step of the run R, which is owned by the agent A playing the initiator role, talks to the responder B, and stores the nonce Nb in its frame. An event's action typically extends the run frame with additional atomic messages, thereby tracking the run's progress. Informally, we call a run *completed* if there is no event that extends its frame.

Example 3.4 (Abstract authentication protocol, part I). The state Σ_{a1} of our model *a1* contains a single variable, *runs*, defined in (1), which models threads executing protocol roles. Since our setup provides initiator and responder roles, this already establishes Requirement **R1** that the protocol has two roles. Each role will generate a nonce that the other role records.

Let na and nb be arbitrary natural numbers. The three events of our specification *a1* abstractly model a protocol that follows a standard challenge-response pattern. The first event, which refines *skip*, just creates an initiator run Ra with an empty frame. The event also "generates" a nonce $Na = Ra\$\mathsf{na}$ associated with this run. Since Na is derived from the run identifier Ra, there is no need to record it in the frame.

```
$step1_{a1}(Ra, A, B, Na) \equiv \{(s, s') \mid$             -- by $A$, refines $skip$
   -- guards
   $Ra \notin dom(s.runs) \wedge$                             -- fresh run id
   $Na = Ra\$\mathsf{na} \wedge$                                -- fresh nonce

   -- actions
   $s'.runs := s.runs(Ra \mapsto (\mathsf{Init}, A, B, []))\ \}.$
```

The second event refines $running_{a0i}$ and creates a responder run identified by Rb and the nonce Nb. The run acquires an *arbitrary* nonce Na, which need not come from an initiator, and records it in its frame. This reflects that the intruder can fake the challenge nonce in later refinements.

```
$step2_{a1}(Rb, A, B, Na, Nb) \equiv \{(s, s') \mid$         -- by $B$, refines $running_{a0i}$
   -- guards
   $Rb \notin dom(s.runs) \wedge$                             -- fresh run id
   $Nb = Rb\$\mathsf{nb} \wedge$                                -- fresh nonce

   -- actions
   $s'.runs := s.runs(Rb \mapsto (\mathsf{Resp}, A, B, [Na]))\ \}.$
```

The final event of the model *a1* will be presented in the next example.

Agents communicate by reading their peers' memories. This is achieved by *non-local guards* that refer to another run's store. Such guards read remote values that may be compared with local values and used in local state updates. We have two kinds of non-local guards: *authorization guards* for secrecy and *authentication guards* for agreements. Authorization guards prevent unauthorized agents from learning secrets. We will explain the shape of these guards in Section 3.4.3 below.

An authentication guard for a given list of agents h and data d (cf. Section 3.3) executed by a run R requires the existence of a run R' executing a different role from R's and agreeing with R on data d, provided the agents in h are honest. For example, the authentication guard

$$A \in good \land B \in good \to \exists Rb.\ Nb = Rb\$\mathsf{nb} \land s.runs(Rb) = (\mathsf{Resp}, A, B, [Ra\$\mathsf{na}])$$

expresses that there exists a run Rb that agrees with the present run Ra on $Ra\$\mathsf{na}$ and Nb, provided A and B are honest (see also Example 3.5 below). More generally, for A in role r to agree with B in role r' on data d assuming honest agents $h = [A, B, \ldots]$, we add an authentication guard G to an appropriate event of agent A (for example, the final event of its role). This guard requires that the agent B in role r' knows the data d, whenever the agents in h are honest. It has the form

$$G(h, d) = h \subseteq good \to \exists R, \overline{x}.\ C(R, d) \land s.runs(R) = (\mathsf{r}', A', B', \ell),$$

where R is the identifier of a run of B in role r', A' and B' are agent names possibly including A and B, ℓ is a list of atomic messages including those in d, and $C(R, d)$ is a conjunction of equations fixing nonces and session keys in d generated by the run R. The free variables of G are exactly the variables appearing in h and d. All other variables are bound by the existential quantification over $\overline{x}$.

Since authorization and authentication guards are related to security properties, we also call them *security guards*. There are also local security guards, which do not refer to other runs' local store. For example, such a guard may check the validity of timestamps to achieve recentness. Note that at this level of abstraction, the intruder is passive and *leak* is the only intruder event of the guard protocol models.

Example 3.5 (Abstract authentication protocol, part II). The third step refines $commit_{a0i}$ and models the initiator run Na receiving its nonce back from a responder run Nb. The first two guards state that the run Ra has not yet received the responder B's nonce and has generated Na. The third guard is an authentication guard that ensures an agreement with the responder on the pair of nonces (Na, Nb).

```
$step3_{a1}(Ra, A, B, Na, Nb) \equiv \{(s, s') \mid$          -- by $A$, refines $commit_{a0i}$
    -- guards
    $s.runs(Ra) = (\mathsf{Init}, A, B, []) \land$
    $Na = Ra\$\mathsf{na} \land$
    $(A \notin bad \land B \notin bad \to \exists Rb.\ Nb = Rb\$\mathsf{nb} \land s.runs(Rb) = (\mathsf{Resp}, A, B, [Na])) \land$

    -- actions
    $s'.runs := s.runs(Ra \mapsto (\mathsf{Init}, A, B, [Nb]))\ \}$.
```

More precisely, if A or B is dishonest then Nb is arbitrary. Otherwise, there is a run Rb of responder B with initiator A that has generated the nonce Nb and previously received the nonce Na. This can be seen as A reading Nb from B's store. We will eliminate this abstraction in the next refinement when we introduce communication channels.

3.4.3 Refinements

We establish the secrecy and authentication properties of our guard protocol models by refining appropriately instantiated secrecy and authentication models from Section 3.3. Below we give general patterns for establishing secrecy and authentication properties. In each case, we establish a refinement by reconstructing the abstract state (i.e., the knowledge and authorization relations kn and az or signals $sigs$) from the concrete run state in terms of the functions knC and azC, or $sigsC$. The concrete definitions of knC, azC, and $sigsC$ depend on the protocol. Moreover, for each such refinement, we identify a pair of concrete protocol events that refine the abstract ones (i.e., secret generation/learning and running/commit, respectively). The remaining events refine *skip*.

Secrecy We establish secrecy by refining the model *s0*. We therefore define functions knC and azC, which reconstruct the knowledge and authorization relations, kn and az, of *s0* from protocol runs, and a function lkC, which abstracts the variable *leak* to the set *lk* of *s0*. Typically, lkC is the identity function (if *leak* just mirrors *lk*) or the domain of a relation (in case *leak* is a relation recording some context information with the leaked data). The simulation relation R_{s01} is π_{s01}^{-1}, where π_{s01} is the mediator function defined as follows.

$$\pi_{s01}(t) \equiv (\!|\, kn = knC(t.runs), az = azC(t.runs), lk = lkC(t.leak) \,|\!).$$

We can now explain how authorization guards are stated in terms of azC and lkC: we use the expression

$$d \notin lkC(t.leak) \rightarrow (d, A) \in azC(t.runs)$$

to check whether an agent A is authorized to access data d whenever d has not leaked to the intruder. We will use authorization guards in Section 4.4 to model the confidential distribution of session keys.

Authentication We similarly refine *a0i* and *a0n* by reconstructing a signal multiset from the concrete runs. The simulation relation R_{a01} is π_{a01}^{-1}, where the mediator function π_{a01} is defined by

$$\pi_{a01}(t) \equiv (\!|\, sigs = sigsC(t.runs) \,|\!).$$

In general, for an agreement of agent A in role r with B in role r' on data d with respect to agents $h = [A, B, \ldots]$, the multiset $sigsC(t.runs)$ contains a $\mathsf{Commit}(h, d)$ signal for each run of A in role r where the data d is known and a $\mathsf{Running}(h, d)$ signal for each run of B in role r' knowing d.

In contrast to secrecy and authentication, we will formulate key freshness as a state predicate and establish it as an invariant of guard protocols (see Section 4.4.4).

Example 3.6 (Refinement of authentication model). Let *a1* be the model from Examples 3.4 and 3.5. The simulation relation R_{01} and mediator function π_{01} are as described for R_{a01} and π_{a01} above. It remains to define the function $sigsC$, which maps completed initiator and responder runs to Commit and $\mathsf{Running}$ signals to express agreement on the nonces Na and Nb. Concretely, the function $sigsC(r)$ translates a run map r (such as *runs* in *a1*) to the multiset of signals (such as *sigs* in *a0i*) that is defined as follows.

$$sigsC(r)(S) = \begin{cases} 1 & \text{if } S = \mathsf{Commit}([A, B], (Ra\$\mathsf{na}, Nb)) \text{ and } r(Ra) = (\mathsf{Init}, A, B, [Nb]), \\ 1 & \text{if } S = \mathsf{Running}([A, B], (Na, Rb\$\mathsf{nb})) \text{ and } r(Rb) = (\mathsf{Resp}, A, B, [Na]), \\ 0 & \text{otherwise.} \end{cases}$$

Note that, in general, the abstraction function $sigsC(r)$ maps each signal to the number of runs in a state that give rise to that signal (see Section 4). Whenever the run identifier appears in the signal's data (as in both cases above), there is at most one such run, since r is a partial function. Therefore, we directly use the constant 1 in the definition above instead of the more complex general definition.

At this point we can state and prove the refinement result, which establishes the entity authentication requirement **R2**. Its proof does not require auxiliary invariants, but relies on a number of basic properties of the function $sigsC$.

Proposition 3.7. $\mathit{a1} \sqsubseteq_{R_{01},\pi_{01}} \mathit{a0i}$.

We have now satisfied both system requirements: injective agreement (**R2**) between an initiator and a responder (**R1**). By using abstraction, we have captured the essential features of entity authentication protocols and established their main property once and for all. Recall that the channel protocols *a2* and *c2* are both refinements of *a1* (cf. Figure 2). Our proofs avoid the intricacies of an active attacker controlling communication. However, the resulting model is still quite abstract and requires further refinement to be executable in the intended hostile distributed environment.

3.5 Level 2 – Channel protocols

At Level 2 of our refinement strategy, we introduce protocols that use communication channels with associated security properties. These channels carry plain text messages without cryptographic operations. We also introduce an active intruder acting in this distributed environment.

3.5.1 Channel messages

For informal use, we adopt the notation of [46] (second column of Table 2). We write $A \rightarrow B$ for an insecure channel from agent A to agent B. The "$: M$" indicates that the message M is sent on the channel. Security properties are indicated by a dot on one or both sides of the arrow. The respective agent has exclusive access to the marked end. A *confidential* channel $A \rightarrow\!\!\bullet\, B$ provides a service to A: A knows that only B can receive messages. An *authentic* channel $A \bullet\!\!\rightarrow B$ provides a service to B: B knows that only A can send messages. A *secure* channel $A \bullet\!\!\rightarrow\!\!\bullet\, B$ provides both guarantees. Besides these static channels, we also use dynamic channels, which are accessed using a key K. For example, $A \bullet\!\!\xrightarrow{K} B$ denotes a dynamic authentic channel. These keys are usually session keys generated during protocol execution. Hence, the associated channels are dynamically created. Static and dynamic channels will later be realized by cryptographic operations using long-term and session keys, respectively.

We formalize the *channel messages* that can be transmitted by a data type *chmsg*, with constructors for static and dynamic channels. The first parameter of these constructors specifies the set of security properties as a combination of authenticity (auth) and confidentiality (confid). The actual payload message is a list of atomic messages.

$$\begin{aligned} \mathit{security} &\triangleq \mathcal{P}\{\mathsf{auth}, \mathsf{confid}\}), \\ \mathit{chmsg} &\triangleq \mathsf{StatCh}(\mathit{security}, \mathit{agent}, \mathit{agent}, [\mathit{atom}]) \mid \mathsf{DynCh}(\mathit{security}, \mathit{key}, [\mathit{atom}]). \end{aligned}$$

Table 2: Channel notation, messages, and conditions for extraction and faking

Channel type	Dot notation	Channel message	Eavesdrop	Fake
insecure	$A \rightarrow B : M$	$\mathsf{Insec}(A, B, M)$	true	true
confidential	$A \rightarrow\!\!\bullet\, B : M$	$\mathsf{Confid}(A, B, M)$	A or B bad	true
authentic	$A \bullet\!\!\rightarrow B : M$	$\mathsf{Auth}(A, B, M)$	true	A or B bad
secure	$A \bullet\!\!\rightarrow\!\!\bullet\, B : M$	$\mathsf{Secure}(A, B, M)$	A or B bad	A or B bad
confidential	$A \xrightarrow{K}\!\!\bullet\, B : M$	$\mathsf{dConfid}(K, M)$	K extractable	K run-bounded
authentic	$A \bullet\!\!\xrightarrow{K} B : M$	$\mathsf{dAuth}(K, M)$	true	K extractable
secure	$A \bullet\!\!\xrightarrow{K}\!\!\bullet\, B : M$	$\mathsf{dSecure}(K, M)$	K extractable	K extractable

Static channel messages name the sender and the receiver. For dynamic channel messages, names are replaced by a key, which determines access to the respective channel. Therefore, the agent names in the informal dot notation for dynamic channels (e.g., in $A \xrightarrow{K}\!\!\bullet\, B$) only suggest the intended communication partners.

In our formal developments, we use the abbreviations given in the third column of Table 2. For example, we define

$$\mathsf{Secure}(A, B, M) \equiv \mathsf{StatCh}(\{\mathsf{auth}, \mathsf{conf}\}, A, B, M)$$

and call this a secure message from A to B. We also say that M is sent to B on a secure channel. We introduce analogous notions for the other channel messages.

3.5.2 Channel-based intruder

Based on the security attributes of channel messages, we define the intruder capabilities for *eavesdropping* (or *extr*acting) payload messages and for *faking* channel messages, indicated in the final two columns of Table 2. We formalize these intruder capabilities as two functions:

$$\begin{aligned} \mathit{extr}_T &: \mathcal{P}(\mathit{chmsg}) \rightarrow \mathcal{P}(\mathit{atom}), \\ \mathit{fake}_{T,U} &: \mathcal{P}(\mathit{chmsg}) \rightarrow \mathcal{P}(\mathit{chmsg}), \end{aligned}$$

where the parameter $T \subseteq \mathit{atom}$ specifies the intruder's initial knowledge and the parameter U denotes a set of run identifiers. The expression $\mathit{extr}_T(H)$ denotes the set of atoms that the intruder can extract from the set of (observed) messages H and $\mathit{fake}_{T,U}(H)$ denotes the set of messages that the intruder can construct from messages in H. The faked messages carry payloads that are extracted from messages in H using $\mathit{extr}_T(H)$. The set of run identifiers U restricts the keys that the intruder can use to fake dynamic channel messages. These functions are defined by the rules in Figures 3 and 4. These rules state that the intruder can eavesdrop messages on non-confidential (i.e., insecure and authentic) channels and fake messages on non-authentic (i.e., insecure and confidential) channels. Moreover, the intruder can eavesdrop messages on confidential channels and fake messages on authentic channels, if these channels have a dishonest starting or ending point (static case) or the associated key K is known to the intruder (dynamic case).

The mild technical condition $K \in \mathit{rkey}(U)$ in the third rule of Figure 4 restricts the intruder to using a session key from the set $\mathit{rkey}(U) \equiv \{R\$i \mid R \in U \wedge i \in \mathbb{N}\}$ to fake a non-authentic

$$\frac{\cdot}{T \subseteq \mathit{extr}_T(H)}$$

$$\frac{\mathsf{StatCh}(c, A, B, M) \in H \quad \mathsf{confid} \notin c \lor A \in \mathit{bad} \lor B \in \mathit{bad}}{M \subseteq \mathit{extr}_T(H)}$$

$$\frac{\mathsf{DynCh}(c, K, M) \in H \quad \mathsf{confid} \notin c \lor K \in \mathit{extr}_T(H)}{M \subseteq \mathit{extr}_T(H)}$$

Figure 3: Rules defining extractable atoms.

$$\frac{\cdot}{H \subseteq \mathit{fake}_{T,U}(H)}$$

$$\frac{M \subseteq \mathit{extr}_T(H) \quad \mathsf{auth} \notin c \lor A \in \mathit{bad} \lor B \in \mathit{bad}}{\mathsf{StatCh}(c, A, B, M) \in \mathit{fake}_{T,U}(H)}$$

$$\frac{M \subseteq \mathit{extr}_T(H) \quad (\mathsf{auth} \notin c \land K \in \mathit{rkey}(U)) \lor K \in \mathit{extr}_T(H)}{\mathsf{DynCh}(c, K, M) \in \mathit{fake}_{T,U}(H)}$$

Figure 4: Rules defining fakeable channel messages.

dynamic message. We call these keys *run-bounded*. Below, we will use $U = \mathit{dom}(s.\mathit{runs})$ to preserve the invariant that all identifiers $R\$i$ with $R \notin \mathit{dom}(s.\mathit{runs})$ are indeed fresh.

The astute reader may be surprised that the intruder does not need to know a key to read from an authentic dynamic channel or write to a confidential dynamic channel. This situation differs from Dolev-Yao-style perfect cryptography. For example, verifying a MAC or signature requires a (shared or public) key. However, the key is typically only required for verifying the authenticity of the message and not for giving access to the authenticated message itself. Likewise, producing an encryption in a Dolev-Yao model requires a key. However, there are encryption schemes such as stream ciphers where the intruder can produce valid ciphertexts without knowing the encryption key. Similarly, it may be surprising that we allow the intruder to (i) eavesdrop messages on confidential channels with a dishonest sender and (ii) send messages on authentic channels with a dishonest receiver. Disallowing these behaviors would preclude implementations of these channels using symmetric cryptography, for instance, realizing an authentic channel using MACs.

3.5.3 State, events, and refinement

Channel protocols extend the state of the guard protocol they refine with a variable *chan* containing a set of channel messages.

$$\Sigma_2 \triangleq \Sigma_1 + (\!|\ \mathit{chan} \in \mathcal{P}(\mathit{chmsg})\ |\!). \tag{2}$$

The protocol events use guards of the form $M \in s.\mathit{chan}$ to receive a channel message M. These guards replace the non-local security guards in the guard protocols, which directly read other runs' local stores. Sending a message M is achieved by an action of the form $s'.\mathit{chan} := s.\mathit{chan} \cup \{M\}$.

Channel protocols include an active intruder event, which closes the set of channel messages under fakeable messages.

$$\mathit{fake}_2 \equiv \{(s, s') \mid s'.\mathit{chan} := \mathit{fake}_{\mathit{ik}_0, \mathit{dom}(s.\mathit{runs})}(s.\mathit{chan})\}. \tag{3}$$

Here, we work with the initial knowledge ik_0 consisting of the sets of all agents, corrupted keys, and numbers.

$$\mathit{ik}_0 \equiv \mathit{agent} \cup \mathit{corrKey} \cup \mathbb{N}$$

The refinement of the abstract Level 1 model just extends the state record with the variable *chan*. Hence, the simulation relation is typically $R_{12} \equiv \Pi_{\mathit{runs}}$, stating that the values of variable *runs* of this and the previous model are identical, or $R_{12} \equiv \Pi_{\mathit{runs},\mathit{leak}}$ in case the state includes an additional variable *leak*.

Example 3.8 (**A channel-based authentication protocol**)**.** The high level of abstraction of guard protocols allows many different realizations. Recall from the overview in Section 3.1 that we have also refined the model *a1* into two protocols, one using authentic and one using confidential channels, respectively. As an example, we now model the following abstract protocol using authentic channels.

M1. $A \to B :\ Na$
M2. $B \bullet\!\!\rightarrow A :\ Nb, Na.$

The initiator A sends the nonce Na to B, who returns it together with his own nonce Nb on an authentic channel. In Section 3.6, we will refine this protocol into the ISO/IEC 9798-3 two-pass unilateral authentication protocol [36].

The state Σ_{a2} of model *a2* is exactly as Σ_2 defined in (2). Initially, all fields are empty. The observation function is π_{runs}, i.e., it projects this state to the *runs* field.

The intruder event fake_{a2} is fake_2 defined in (3). The protocol events send and receive messages to and from the insecure and authentic channels. In the first step, $\mathit{step1}_{a1}$, the initiator A sends Na on an insecure channel to B. In the second step, the responder B creates a new run identified by Rb, an associated nonce Nb, receives message M1 from the insecure channel, and authentically sends message M2 to A. This event adds the reception and sending of messages to the event $\mathit{step2}_{a1}$ that it refines. In particular, instead of accepting any nonce Na, the nonce is now extracted from M1.

```
$\mathit{step2}_{a2}(Rb, A, B, Na, Nb) \equiv \{(s, s') \mid$          -- by $B$, refines $\mathit{step2}_{a1}$
   -- guards
   $Rb \notin \mathit{dom}(s.\mathit{runs}) \wedge$                   -- fresh run $Rb$
   $Nb = Rb\$\mathsf{nb} \wedge$                                      -- fresh nonce $Nb$
   $\mathsf{Insec}(A, B, [Na]) \in s.\mathit{chan} \wedge$            -- receive M1

   -- actions
   $s'.\mathit{runs} := s.\mathit{runs}(Rb \mapsto (\mathsf{Resp}, A, B, [Na])) \wedge$
   $s'.\mathit{chan} := s.\mathit{chan} \cup \{\mathsf{Auth}(B, A, [Nb, Na])\}\,\}.$   -- send M2
```

In the third step, the initiator run Ra receives message M2 and updates his local state with the nonce Nb.

$$
\begin{array}{ll}
\mathit{step3}_{a2}(Ra, A, B, Na, Nb) \equiv \{(s, s') \mid & \texttt{-- by } A\texttt{, refines } \mathit{step3}_{a1} \\
\quad \texttt{-- guards} & \\
\quad s.\mathit{runs}(Ra) = (\mathsf{Init}, A, B, []) \land & \\
\quad Na = Ra\$\mathsf{na} \land & \\
\quad \mathsf{Auth}(B, A, [Nb, Na]) \in s.\mathit{chan} \land & \texttt{-- recv M2} \\
\quad \texttt{-- actions} & \\
\quad s'.\mathit{runs} := s.\mathit{runs}(Ra \mapsto (\mathsf{Init}, A, B, [Nb]))\,\}. &
\end{array}
$$

The reception of message M2 replaces the access to the responder run's memory of the refined event $\mathit{step3}_{a1}$. The corresponding guard refinement **(GRD)** of the refinement proof requires the following invariant, which states that authentic messages between honest agents indeed originate from an associated responder run identified by Rb.

$$
\begin{array}{l}
\mathit{auth}_{a2} \equiv \{s \mid \forall A, B, Na, Nb. \\
\quad \mathsf{Auth}(B, A, [Nb, Na]) \in s.\mathit{chan} \land B \notin \mathit{bad} \land A \notin \mathit{bad} \\
\quad \to \exists Rb.\ Nb = Rb\$\mathsf{nb} \land s.\mathit{runs}(Rb) = (\mathsf{Resp}, A, B, [Na])\}.
\end{array}
$$

This invariant is all that is needed to prove the refinement. Note that this is an internal invariant since it refers to channel messages, which are not observable.

Let $a2$ be the above specification. The simulation relation is the canonical $R_{12} \equiv \Pi_{\mathit{runs}}$ described above.

Proposition 3.9. *Let $R'_{12} \equiv R_{12} \cap (\Sigma_{a1} \times \mathit{auth}_{a2})$. Then we have $\mathit{reach}(a2) \subseteq \mathit{auth}_{a2}$ and $a2 \sqsubseteq_{R'_{12},id} a1$.*

By using abstract channels, we retain the possibility of different cryptographic realizations. For example, we may realize the authentic channels using signatures or MACs.

3.6 Level 3 – Cryptographic protocols

We model concrete protocols and the Dolev-Yao intruder using a standard theory of *cryptographic messages* due to Paulson [59]. At this level, messages are transmitted over insecure channels.

3.6.1 Cryptographic messages and setup

The type of messages, msg, is defined inductively from agents $A \in \mathit{agent}$, nonces $N \in \mathit{nonce}$, timestamps $T \in \mathbb{N}$, keys $K \in \mathit{key}$, pairs $\langle M_1, M_2 \rangle$, and encryptions $\{\!|M|\!\}_K$. We sometimes omit the pair brackets $\langle \cdot, \cdot \rangle$ when no confusion can result. At this level, we can define a concrete type of long-term keys, ltk. As one of many possible examples, we may define ltk as follows.

$$
\mathit{ltk} \triangleq \mathsf{pub}(\mathit{agent}) \mid \mathsf{pri}(\mathit{agent}) \mid \mathsf{shr}(\mathit{agent}).
$$

The terms $\mathsf{pub}(A)$, $\mathsf{pri}(A)$, and $\mathsf{shr}(A)$ respectively denote A's public key, A's private key, and the symmetric key that A shares with the server S. We model signatures as encryptions with a private key.

We can also concretize the previously declared abstract setup of cryptographic keys, *keySetup* (see Section 3.2). For example, we may define *keySetup* as follows.

$$\frac{A, B \in \mathit{agent}}{(\mathsf{pub}(A), B) \in \mathit{keySetup}} \qquad \frac{A \in \mathit{agent}}{(\mathsf{pri}(A), A) \in \mathit{keySetup}} \qquad \frac{A \in \mathit{agent} \quad C \in \{A, \mathsf{S}\}}{(\mathsf{shr}(A), C) \in \mathit{keySetup}}.$$

As a consequence we can prove the following equation about *corrKey*.

$$\mathit{corrKey} = \mathsf{pub}(\mathit{agent}) \cup \mathsf{pri}(\mathit{bad}) \cup \mathsf{shr}(\mathit{bad}).$$

To formalize protocol properties and the intruder, we use the standard closure operators *parts*, *analz*, and *synth* on sets of messages (see [59]; for completeness, we give their definitions in Appendix A). Note that our framework is currently restricted to encryption with atomic keys. The term $\mathit{parts}(H)$ closes H under submessages (i.e., subterms of messages), $\mathit{analz}(H)$ closes H under submessages accessible by projection and decryption using the keys in H, and $\mathit{synth}(H)$ closes H under message compositions.

3.6.2 State and events

Cryptographic protocols replace the channel messages *chan* with a variable *IK* containing a set of cryptographic messages. Therefore, like the channel protocols, they extend the state Σ_1 of the guard protocol they refine.

$$\Sigma_3 \triangleq \Sigma_1 + (\!| \mathit{IK} \in \mathcal{P}(\mathit{msg}) |\!).$$

Initially, the set *IK* contains the initial intruder knowledge, e.g., the long-term keys of all bad agents, *corrKey*. Note that $\mathit{agent} \cup \mathbb{N} \subseteq \mathit{synth}(H)$ for all sets of messages H.

Protocol events receive messages by using guards of the form $M \in s.\mathit{IK}$ and send messages by actions of the form $s'.\mathit{IK} := s.\mathit{IK} \cup \{M\}$. The Dolev-Yao intruder can generate and send messages from the set $\mathit{synth}(\mathit{analz}(s.\mathit{IK}))$.

$$\mathit{fake}_3 \equiv \{(s, s') \mid s'.\mathit{IK} := \mathit{synth}(\mathit{analz}(s.\mathit{IK}))\}$$

Example 3.10 (A cryptographic authentication protocol). We now refine the abstract protocol model *a2* into the ISO/IEC 9798-3 two-pass unilateral authentication protocol [36] by translating the authentic channels into signed messages communicated over an insecure channel.

M1. $A \to B : A, B, Na$
M2. $B \to A : \{\!| Nb, Na, A |\!\}_{\mathsf{pri}(B)}$.

The state Σ_{iso3} is Σ_3 described above. Initially, the *runs* field is empty and $\mathit{IK} = \mathit{corrKey}$. Note that for this example the shared keys could be omitted. This models Assumptions **A2** and **A3** about static corruption and the cryptographic setup. Only the *runs* field is observable.

We restrict our presentation to the second protocol step, $\mathit{step2}_{\mathit{iso3}}$. It refines the abstract event $\mathit{step2}_{\mathit{a2}}$ by replacing the insecure message M1 by a corresponding message in *IK* and the authentic message M2 by a signed message in IK.

```
$step2_{iso3}(Rb, A, B, Na, Nb) \equiv \{(s, s') \mid$          -- by $B$, refines $step2_{a2}$
  -- guards
  $Rb \notin dom(s.runs) \land$
  $Nb = Rb\$\mathsf{nb} \land$
  $\langle A, B, Na \rangle \in s.IK \land$                     -- receive M1

  -- actions
  $s'.runs := s.runs(Rb \mapsto (\mathsf{Resp}, A, B, [Na])),$
  $s'.IK := s.IK \cup \{\{\!|Nb, Na, A|\!\}_{\mathsf{pri}(B)}\}\}.$          -- send M2
```

The intruder event $fake_{iso3}$ is $fake_3$ introduced above, which models the Dolev-Yao intruder described in Assumption **A1**.

3.6.3 Refinement

The refinement of channel protocols by cryptographic ones is based on a protocol-dependent message abstraction function $absMsg : \mathcal{P}(msg) \to \mathcal{P}(chmsg)$. Given such a function, the simulation relation R_{23} is defined as the intersection of the following four relations.

$$
\begin{aligned}
R_{23}^{msgs} &\equiv \{(s,t) \mid absMsg(parts(t.IK)) \subseteq s.chan\},\\
R_{23}^{non} &\equiv \{(s,t) \mid \forall N \in nonce.\ N \in analz(t.IK) \to N \in extr_{ik_0}(s.chan)\},\\
R_{23}^{key} &\equiv \{(s,t) \mid \forall K \in key.\ K \in analz(t.IK) \to K \in extr_{ik_0}(s.chan)\},\\
R_{23}^{pres} &\equiv \Pi_{runs}.
\end{aligned}
\tag{4}
$$

The relation R_{23}^{msgs} expresses that the abstractions of concrete message parts in $t.IK$ are contained in the channel variable $s.chan$. Abstracting message parts instead of the messages themselves increases the flexibility of the abstractions. We will further discuss this point in Section 4.6 where this possibility enables the modification of the communication topology between Levels 2 and 3. The relations R_{23}^{key} and R_{23}^{non} state that the abstract intruder knows at least the nonces and keys that the concrete intruder also knows. If no keys or no nonces appear in a protocol then the corresponding relation can be dropped from the simulation relation. Finally, the relation R_{23}^{pres} states that the variable $runs$ is preserved, i.e., it has the same value in the abstract and concrete model. In concrete applications, this relation may include other preserved state variables, such as $leak$.

When modeling session key compromise, it may be necessary to generalize this simulation relation, in particular the relations R_{23}^{key} and R_{23}^{non}. We defer the detailed discussion of this case to Section 4.6.

In the refinement proof for the intruder events, a variant of the following action refinement **(ACT)** proof obligation arises.

$$(s(\!|\ chan := fake(s.chan)\ |\!),\ t(\!|\ IK := synth(analz(t.IK))\ |\!)) \in R_{23}.$$

This states that the successor states resulting from the respective intruder actions are still in the simulation relation. The proof of the part concerning R_{23}^{msgs} relies on the property

$$(s,t) \in R_{23} \ \to\ absMsg(parts(synth(analz(t.IK)))) \subseteq fake(s.chan),$$

which typically follows from the definitions of $absMsg$ and $fake$ and from general properties of $parts$, $synth$, and $analz$.

Example 3.11. The simulation relation in this refinement is R_{23} as defined in (4), except that R_{23}^{key} is not needed. We concretize the message abstraction function $absMsg(H)$ as follows.

$$\frac{\langle A, B, Na\rangle \in H}{\mathsf{Insec}(A, B, [Na]) \in absMsg(H)} \qquad \frac{\{\!|Nb, Na, A|\!\}_{\mathsf{pri}(B)} \in H}{\mathsf{Auth}(B, A, [Nb, Na]) \in absMsg(H)}.$$

The refinement proof only requires one internal invariant expressing the secrecy of the private signing keys.

$$keys_{iso3} \equiv \{s \mid \forall A.\ pri(A) \in analz(s.IK) \rightarrow A \in bad\}.$$

We can now state the refinement result relating this model of the ISO/IEC 9798-3 protocol to the abstract channel model *a2*.

Proposition 3.12. *Let* $R'_{23} \equiv R_{23} \cap \Sigma_{a2} \times keys_{iso3}$*. Then* $reach(iso3) \subseteq keys_{iso3}$ *and* $iso3 \sqsubseteq_{R'_{23},id} a2$*.*

This concludes our running example.

3.7 Discussion

We have presented our four-level refinement strategy along with its supporting infrastructure. We have illustrated our approach with the development of simple entity authentication protocols. This resulted in two different concrete protocols at Level 3: ISO/IEC 9798-3 and NSL/2. We satisfied all system requirements by proving properties of the abstract model at Levels 0 and 1. As these models are not directly implementable, we continued our refinements, thereby obtaining models that are suitable for an implementation in the intended hostile distributed environment and, crucially, inherit the properties we proved for the abstract models. The simulation relation and invariants used here at Levels 2 and 3 are canonical for our refinement strategy (cf. Section 4). We would like to emphasize that a number of alternative cryptographic realizations of the channel protocols are possible, for example, using symmetric encryption or MACs. Our approach fosters abstraction and enables the sharing of structure and proofs.

4 Key establishment protocols

In this section, we validate our refinement approach by developing a family of key establishment protocols, including the Needham-Schroeder Shared Key (NSSK) protocol, core versions of the Kerberos 4 and 5 protocols, and the Denning-Sacco protocol. Compared to the running example from Section 3, our protocol models feature additional elements such as timestamps, replay caches, dynamic channels, and a changing communication structure. We also prove additional security properties related to session keys, such as key confirmation, key freshness, key recentness, and session key compromise.

4.1 Requirements and assumptions

Our informal requirements and assumptions for (server-based) key establishment protocols follow below. The first three requirements are mandatory and must be satisfied by all protocols we consider. The last three requirements are optional. We will formalize these requirements in subsequent sections.

Requirement R1 (Key distribution). The server generates and distributes a fresh session key to an initiator and a responder.

Requirement R2 (Key secrecy). Only authorized agents may learn a session key K, unless one of them is dishonest or the key K has leaked whereby other agents may also learn it.

The next two requirements cover authentication properties, which we will formalize in Section 4.3 as injective or non-injective agreements.

Requirement R3 (Server authentication). The initiator and the responder each authenticate the server on the session key and possibly on additional data.

Requirement R4 (Key confirmation). The initiator and the responder authenticate each other on the session key and possibly on additional data, thereby confirming to each other their knowledge of the key.

Two additional (and independent) requirements concern the freshness and recentness of the session key. A key is *fresh* if it is only used in a single session and is *recent* if its lifetime does not exceed a specified limit.

Requirement R5 (Key freshness). The initiator and responder obtain assurance that the session key is fresh.

Requirement R6 (Key recentness). The initiator and responder obtain assurance that the session key is recent.

The environment assumptions **A1**–**A3** about the intruder, static corruption, and the cryptographic setup remain the same as in Section 3.1. We also add the possibility of session key compromise.

Assumption A4 (Session key compromise). Session keys may leak to the intruder.

4.2 Development overview

We concretize our refinement strategy for deriving different server-based key establishment protocols: the Needham-Schroeder Shared-Key (NSSK) protocol [54], the Denning-Sacco protocol [32], and a core version of Kerberos 4 [66] and Kerberos 5 [55] with one instead of two servers. In these protocols, the initiator requests a session key from the server for use with a given responder. We derive different variants of these protocols. In the simplest variant, the server responds by sending encrypted copies of the session key directly to the initiator and the responder. In the original protocols, the server also sends the responder's encrypted key, called a *ticket*, to the initiator who forwards it to the responder. The ticket is either encrypted inside the message containing the initiator's copy of the session key (NSSK, Denning-Sacco, and Kerberos 4) or sent alongside that message (Kerberos 5). Moreover, in the NSSK and Kerberos 4 and 5 protocols, the initiator and the responder exchange two additional messages for mutual key confirmation. After this overview, we will focus on the core Kerberos 4 and 5 protocols in the remainder of this section. Figure 12 on page 40 depicts a message sequence chart of these protocols.

The refinement graph in Figure 5 summarizes our development. Recall that each node represents a model and each arc $m \to m'$ represents a refinement $m \sqsubseteq_\pi m'$ for a mediator

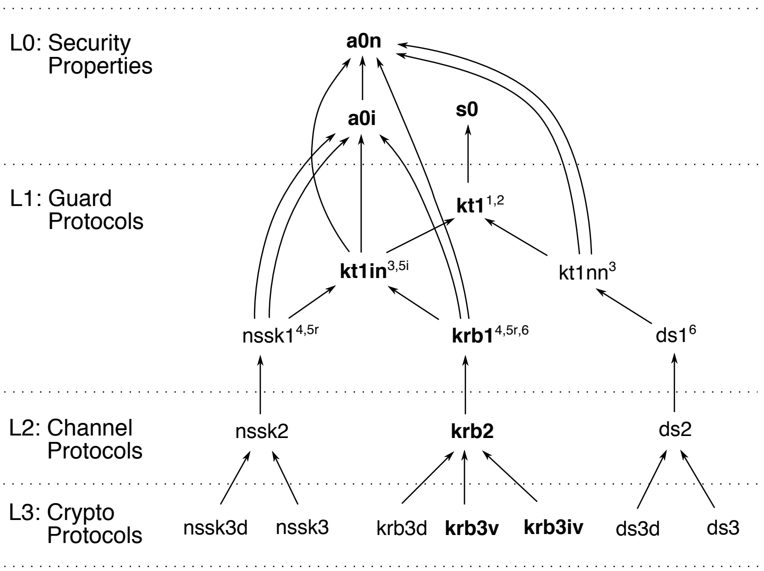


Figure 5: Refinement graph.

function π, not shown. The superscripts refer to the requirements established, where i and r denote the initiator and the responder.

At Level 0, we have the abstract models of secrecy (*s0*) and authentication (*a0i*, *a0n*) from Section 3.3. At Level 1, our first guard protocol, *kt1*, abstractly models server-based secret key transport (**R1**, **R2**). It refines the secrecy model *s0* and is an ancestor of all key transport protocols that we have derived. The model *kt1* therefore provides secret key distribution, but does not guarantee the session key's authenticity, freshness, or recentness. Hence, we refine *kt1* into further guard protocols that establish authentication properties (**R3**, **R4**) and use nonces or timestamps, to prevent replays and guarantee key freshness and recentness (**R5**, **R6**). We do this in two stages.

In the first stage, we refine *kt1* into two different models that realize server authentication (**R3**). In the first model, *kt1in*, the initiator injectively agrees with the server on the session key, while the responder non-injectively agrees with the server (reflected in the name as ending *in*). The injective agreement and secrecy entail key freshness (**R5**) for the initiator. In the second model, *kt1nn*, both initiator and responder achieve non-injective agreements with the server (reflected in the name as *nn*). The data agreed upon, in addition to the session key, is a parameter in these models. We establish each authentication property by refining an L0 model, *a0n* or *a0i*, using a different mediator function. This explains why there are multiple arcs between some models in Figure 5.

In the second stage, we refine the model *kt1in* into *nssk1* and *krb1* and establish key confirmation (**R4**). We achieve this by adding protocol steps and proving mutual agreement between the initiator and the responder on the key and other data. In *nssk1*, these agreements are injective due to the use of nonces. In *krb1*, we use timestamps to ensure key recentness (**R6**). A replay prevention cache allows the responder to obtain an injective agreement with the initiator. Key freshness (**R5**) for the responder relies on both authentication and secrecy properties. Finally, we also refine the model *kt1nn* into *ds1* using timestamps to obtain key recentness. At this point, all requirements are established. The remaining two levels realize the

Table 3: L3 models and their properties

Protocol	Model	**R1**	**R2**	**R3**	**R4**	**R5**	**R6**
NSSK	*nssk3*	✓	✓	i/n	i/i	✓	–
Kerberos 4	*krb3iv*	✓	✓	i/n	n/i	✓	✓
Kerberos 5	*krb3v*	✓	✓	i/n	n/i	✓	✓
Denning-Sacco	*ds3*	✓	✓	n/n	–	–	✓

environment assumptions **A1**–**A4**, making the protocol fit for execution in a hostile distributed environment.

At Level 2, we construct the three channel-based models, *nssk2*, *krb2*, and *ds2*, where the roles exchange channel messages instead of reading each other's memory. The server distributes the session key on static secure channels to the initiator and responder and key confirmation is realized in *nssk2* and *krb2* using dynamic authentic channels protected by the session key.

At Level 3, we replace the channel messages by cryptographic messages sent over an insecure channel. We implement the static secure channels by symmetric encryption with long-term keys and the dynamic authentic channels by encryption with the session key. The models at this level differ in their handling of the responder's ticket. In models with names ending in *d* (for direct), namely, *nssk3d*, *ds3d*, and *krb3d*, the server sends the ticket directly to the responder. In the other models, the communication topology changes: the server sends the ticket to the initiator who forwards it to the responder. While in *krb3v* the ticket is sent alongside the ciphertext containing the initiator's session key, it appears inside the ciphertext in the models *nssk3*, *ds3*, and *krb3iv*.

Table 3 summarizes the requirements achieved by the final protocols. In the columns for the authentication requirements **R3** and **R4**, 'i' and 'n' mean injective and non-injective agreement. The slash separates initiator and responder guarantees. The models with names ending in *d* (not listed in the table) achieve the same properties as their listed siblings.

Based on the modeling and reasoning framework and the infrastructure from Section 3, in the following sections we develop concrete models at each abstraction level. We focus on the models typeset in boldface in Figure 5 that lead to the core versions of Kerberos. Figure 6 provides an overview of most refinements between these models and the related propositions. State spaces (without variable types) and simulation relations are displayed on the left-hand side, observations and mediator functions on the right-hand side, and observation functions are shown as left-to-right arrows. Triples of the form (i, r, s) describe the frames of completed initiator, responder, and server runs, where "..." stands for the fields inherited from the model above. A star (*) means that the related refinement proof requires invariants to strengthen the simulation relation. In this figure, we have not included the refinements of the authentication models *a0n* and *a0i* by the models *kt1in* and *krb1* (Propositions 4.2 and 4.5). These exhibit a structure similar to the refinement of *s0* by *kt1*. We have also omitted the refinement of *krb2* into core Kerberos 4 (*krb3iv*) stated in Proposition 4.9 as it is similar to the refinement into core Kerberos 5 (*krb3v*). This figure is primarily intended as a reference for the reader, but we will return to it in Section 4.7.1 where we discuss the security guarantees that the refinements yield for the final models at Level 3.

$$\begin{tikzcd}
\Sigma_{s0} \triangleq \llparenthesis\, kn, az, lk \,\rrparenthesis \arrow[r, "id"] \arrow[d, "\pi_{s01}^{-1}\ (*)"'] \arrow[dr, phantom, "\boxed{\text{Proposition 4.1}}"] & \Sigma_{s0} \\
\begin{matrix} \Sigma_{kt1} \triangleq \llparenthesis\, runs, leak \,\rrparenthesis \\ ([Kab], [Kab], []) \end{matrix} \arrow[r, "id"] \arrow[d, "\pi_{1in1}^{-1}"'] \arrow[dr, phantom, "\boxed{\text{Proposition 4.3}}"] & \Sigma_{kt1} \arrow[u, "\pi_{s01}"] \\
\begin{matrix} \Sigma_{kt1in} \triangleq \Sigma_{kt1} \\ ([\ldots, Ts], [\ldots, Ts], [Na, Ts]) \end{matrix} \arrow[r, "id"] \arrow[d, "\pi_{11}^{-1}"'] \arrow[dr, phantom, "\boxed{\text{Proposition 4.4}}"] & \Sigma_{kt1} \arrow[u, "\pi_{1in1}"] \\
\begin{matrix} \Sigma_{krb1} \triangleq \llparenthesis\, runs, leak, clk, cch \,\rrparenthesis \\ ([\ldots, Ta], [\ldots, Ta, END], [\ldots]) \end{matrix} \arrow[r, "id"] \arrow[d, "\Pi_{runs,leak,clk,cch}\ (*)"'] \arrow[dr, phantom, "\boxed{\text{Proposition 4.7}}"] & \Sigma_{krb1} \arrow[u, "\pi_{11}"] \\
\begin{matrix} \Sigma_{krb2} \triangleq \Sigma_{krb1} + \llparenthesis\, chan \,\rrparenthesis \\ ([\ldots], [\ldots], [\ldots]) \end{matrix} \arrow[r, "\pi_{runs,leak,clk,cch}"] \arrow[d, "R_{23}\ (*)"'] \arrow[dr, phantom, "\boxed{\text{Proposition 4.8}}"] & \Sigma_{krb1} \arrow[u, "id"] \\
\begin{matrix} \Sigma_{krb3v} \triangleq \Sigma_{krb1} + \llparenthesis\, IK \,\rrparenthesis \\ ([\ldots], [\ldots], [\ldots]) \end{matrix} \arrow[r, "\pi_{runs,leak,clk,cch}"] & \Sigma_{krb1} \arrow[u, "id"]
\end{tikzcd}$$

Figure 6: Details of refinements in the Kerberos development.

4.3 Security properties (L0)

We start our development by formalizing the security requirements. We formalize each secrecy and authentication requirement as an instance of the corresponding L0 model from Section 3.3. We will later show that our guard protocol models (L1) refine these instantiated models, thus establishing the respective requirements (by Corollary 2.8). We will formalize key recentness directly as a guard and key freshness as invariants of Level 1 protocols and therefore discuss these later in Section 4.4.

Secrecy The instantiation of the polymorphic type of data of the model *s0* to keys provides an abstract model of key distribution and key secrecy. Refining this model will establish Requirements **R1** and **R2**.

Authentication We formalize the Requirements **R3** and **R4**. For this purpose, we must specify the data agreed upon. We use *authentication graphs* to represent this information visually. Figure 7 displays the authentication graph of the Kerberos protocols. In these graphs, there is one node for each protocol role. Each node is labeled by an agent name (in the given role), possibly followed by a list of nonces and timestamps generated during the role's execution. For example, the server S generates the session key *Kab* and a timestamp *Ts*, and the initiator generates a nonce *Na* and a timestamp *Ta*. Each arrow specifies one agreement property, guaranteed to the agent at the arrow's head, by defining the parameters *h* and *d* for the *running*

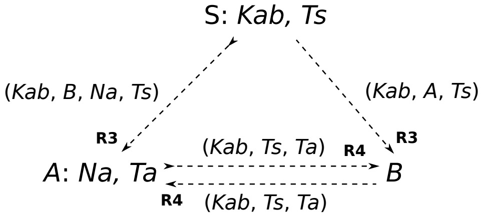


Figure 7: Authentication graph for Kerberos.

and *commit* events of the models *a0n* or *a0i*. The arrow endpoints define the agents h, whose honesty is assumed, and the tuples labeling the arrows specify the data d to be agreed upon between these agents. Note that for the current development, we do not need to assume the honesty of agents other than the participants in the agreement. An arrow tail indicates an injective agreement. The boldface labels indicate the requirements that are established for the agent near the arrow head. For example, the arrow from S to B labeled by (Kab, A, Ts) means that the responder B non-injectively agrees with the server on (Kab, A, Ts), assuming the honesty of S and B (**R3**). The arrow from A to B labeled by (Kab, Ts, Ta) means that B injectively agrees with A on Kab, Ts, and Ta, assuming the honesty of A and B (**R4**).

To prove that an L1 model establishes an agreement of role r with role r', we identify an event of role r that refines the *commit* event and an event of role r' that refines the *running* event. All other events must refine *skip*. Each agreement requires a different mapping of the protocol events to the *running* and *commit* events and therefore requires a separate refinement of the model *a0i* or *a0n* (cf. Figure 5).

4.4 Guard protocols (L1)

In our server-based key transport protocols, there are three roles: a key-generating *server* and key-receiving *initiators* and *responders*. The state records runtime information about the execution of these roles and the set of leaked keys as described in Section 3.4.

$$\begin{aligned} \Sigma_1 &\triangleq (\!|\; runs \in runsT \;|\!), \\ \Sigma_{kt1} &\triangleq \Sigma_1 + (\!|\; leak \in \mathcal{P}(key) \;|\!). \end{aligned}$$

Initially, the *runs* map is empty and *leak* is initialized to the set of corrupted keys, *corrKey*. The entire state is observable, i.e., the observation function is the identity.

4.4.1 Secret key distribution

A sequence chart of our first abstract key transport protocol model, *kt1*, appears in Figure 8. This model establishes **R1** and **R2** as follows. The server S generates the session key Kab, which is indicated by the role label S: Kab. The initiator A and the responder B then *secretly* acquire this key and record it in their run frames, which is represented by arrows from S to A and B labeled by Kab. These arrows do *not* represent the communication of messages, since there are no messages or channels at this stage.

Before presenting the events of the specification *kt1*, we discuss the simulation relation used in the refinement of the model *s0* from Section 3.3.1, which establishes session key secrecy. As described in Section 3.4, we define relations $knC(r)$ and $azC(r)$, which reconstruct *s0*'s

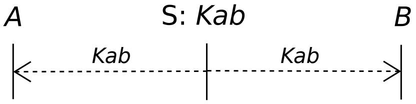


Figure 8: Basic secret key distribution (*kt1*).

knowledge and authorization relations from the runs $r \in runsT$. We also define the function lkC as the identity function, i.e., the variable *leak* equals *lk* in *s0*.

We define the relation $knC(r)$ by four rules. The following three rules describe each role's session key knowledge. Recall that $x\#xs$ is a list with head element x and tail xs.

$$\frac{r(Ra) = (\mathsf{Init}, A, B, K\#ns)}{(K, A) \in knC(r)} \qquad \frac{r(Rb) = (\mathsf{Resp}, A, B, K\#ns)}{(K, B) \in knC(r)} \qquad \frac{r(Rs) = (\mathsf{Serv}, A, B, ns)}{(Rs\$\mathsf{sk}, \mathsf{S}) \in knC(r)}.$$

Here, $Rs\$\mathsf{sk}$ is the fresh value used by the server run Rs for the session key and sk is an arbitrary natural number. An additional rule states that the initial key setup is contained in the knowledge relation, i.e., $keySetup \subseteq knC(r)$.

The following rule defines who is authorized to learn a session key that the server S generated for A and B, namely A, B, and S, if A and B are honest, and everyone otherwise.

$$\frac{r(Rs) = (\mathsf{Serv}, A, B, ns) \qquad C \in \{A, B, \mathsf{S}\} \lor A \in bad \lor B \in bad}{(Rs\$\mathsf{sk}, C) \in azC(r)}. \tag{5}$$

An additional rule states that $keySetup \subseteq azC(r)$.

The specification *kt1* has five events modeling a protocol step and a leak event modeling session key compromise. The first event creates a new run Ra of initiator A with responder B by updating *runs* with $(Ra \mapsto (\mathsf{Init}, A, B, []))$. The second event creates a responder run analogously. These two events refine *skip*. In the third event, we generate a new server run Rs with an associated fresh session key Kab. This event refines $gen_{s0}(Kab, \mathsf{S}, [\mathsf{S}, A, B])$.

```
$step3_{kt1}(Rs, A, B, Kab) \equiv \{(s, s') \mid$           -- by $\mathsf{S}$, refines $gen_{s0}$
    $Rs \notin dom(s.runs) \land$                           -- fresh server run
    $Kab = Rs\$\mathsf{sk} \land$                              -- session key

    $s'.runs := s.runs(Rs \mapsto (\mathsf{Serv}, A, B, []))\}$.
```

The last two events model the acquisition of the session key by the initiator and the responder. They both refine the event $learn_{s0}$. In Step 5, the responder B acquires the session key Kab in its run Rb.

```
$step5_{kt1}(Rb, A, B, Kab) \equiv \{(s, s') \mid$           -- by $B$, refines $learn_{s0}$
    $s.runs(Rb) = (\mathsf{Resp}, A, B, []) \land$                -- $B$'s run
    $(Kab \notin s.leak \to (Kab, B) \in azC(s.runs)) \land$   -- check authorization

    $s'.runs := s.runs(Rb \mapsto (\mathsf{Resp}, A, B, [Kab]))\}$.
```

The first guard requires that Rb identifies a run of responder B with initiator A, where B has not yet received a key. The action updates the responder run. The initiator's $step4_{kt1}$ is analogous.

The second guard is an authorization guard requiring that B is authorized to learn Kab unless Kab is leaked. There are two cases according to the definition of azC. The first case, described by rule (5), corresponds to reading a (session) key Kab from the server, who determined the authorization to access the key. Note that there is no guarantee that the key was generated for B. In the second case, Kab is a static key, which may be corrupted. For now, there are no further constraints on Kab. The authorization guard is sufficient to preserve the secrecy of Kab. In Section 4.4.2, we establish authentication properties to ensure that honest agents only accept session keys generated for them and shared with the intended partner.

Finally, the leak event compromises the session key generated by a server run and records it in the variable *leak*.

$$\begin{array}{ll}
\mathit{leak}_{\mathit{kt1}}(\mathit{Rs}) \equiv \{(s, s') \mid \exists A, B, \mathit{ns}. & \texttt{-- by attacker, refines } \mathit{leak}_{\mathit{s0}} \\
\quad s.\mathit{runs}(\mathit{Rs}) = (\mathsf{Serv}, A, B, \mathit{ns}) \wedge & \texttt{-- existing server run} \\
\quad s'.\mathit{leak} := s.\mathit{leak} \cup \{\mathit{Rs}\$\mathsf{sk}\}\ \}. &
\end{array}$$

Instantiating the simulation relation R_{s01} from Section 3.4 with the relations $azC(r)$ and $knC(r)$ defined above and lkC with the identity function (i.e., $\mathit{lk} = \mathit{leak}$), we show that the model $\mathit{kt1}$ refines the secrecy model $\mathit{s0}$. The guard strengthening proof in the refinement of the event $\mathit{gen}_{\mathit{s0}}$ by $\mathit{step3}_{\mathit{kt1}}$ requires an invariant, $\mathit{key}_{\mathit{kt1}}$, stating that a fresh session key K is neither in the domain of $knC(s.\mathit{runs})$ or $azC(s.\mathit{runs})$ nor an element of $s.\mathit{leak}$.

Proposition 4.1. Let $R'_{\mathit{s01}} \equiv R_{\mathit{s01}} \cap (\Sigma_{\mathit{s0}} \times \mathit{key}_{\mathit{kt1}})$. *Then we have* $\mathit{reach}(\mathit{kt1}) \subseteq \mathit{key}_{\mathit{kt1}}$ *and* $\mathit{kt1} \sqsubseteq_{R'_{\mathit{s01}}, \pi_{\mathit{s01}}} \mathit{s0}$.

Since the abstract variables kn and az are observable and reconstructable from the concrete state, the secrecy invariant for $\mathit{s0}$ (Proposition 3.1) is inherited by $\mathit{kt1}$ (Corollary 2.8), which thus realizes secret key distribution (**R1**, **R2**).

4.4.2 Server authentication

We now refine the model $\mathit{kt1}$ into $\mathit{kt1in}$ and establish agreements of the initiator and the responder with the server on the session key and additional data. The additional data is a parameter of $\mathit{kt1in}$. However, for the sake of the presentation, we will focus our attention on the instantiation of $\mathit{kt1in}$ for the Kerberos development. The models $\mathit{kt1}$ and $\mathit{kt1in}$ have identical state spaces, but in $\mathit{kt1in}$ we introduce nonces and timestamps, which are part of the data included in the agreement and recorded in the run frames of the different roles. The observation function is the identity. Figure 9 shows a sequence chart for this model. The labels below the arrows denote agreements and an arrow tail indicates an injective agreement as specified in Figure 7. Na is a nonce generated by A and Ts is a timestamp generated by S. The initiator A achieves (**R3**) by an injective agreement with the server on $(\mathit{Kab}, B, \mathit{Na}, \mathit{Ts})$ and the responder B establishes (**R3**) by a non-injective agreement with the server on $(\mathit{Kab}, A, \mathit{Ts})$. The model $\mathit{kt1in}$ refines $\mathit{kt1}$, $\mathit{a0i}$, and $\mathit{a0n}$ (cf. Figures 5 and 6). We establish key freshness (**R5**) for the initiator as an invariant (cf. Section 4.4.4 for the similar responder case).

We obtain the model $\mathit{kt1in}$ by modifying $\mathit{kt1}$ in two ways. First, we introduce new event parameters and corresponding state updates to reflect that both partners of the agreement know the data being agreed upon. In the server's Step 3, we add a nonce Na and the timestamp Ts to the parameters and record these in the run frame, i.e., the runs are updated with $\mathit{Rs} \mapsto$

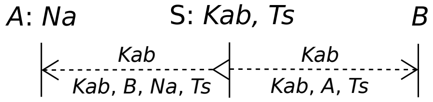


Figure 9: Adding server authentication (*kt1in*).

$(\mathsf{Serv}, A, B, [\mathit{Na}, \mathit{Ts}])$. Neither Na nor Ts are constrained by any guards. In the responder's Step 5, we add the parameter Ts and update the runs with $\mathit{Rb} \mapsto (\mathsf{Resp}, A, B, [\mathit{Kab}, \mathit{Ts}])$ and similarly in the initiator's Step 4.

Second, we realize the agreements described above by adding authentication guards to the key-receiving Steps 4 and 5. These guards may be added directly to the respective events or *discovered* during the refinement proof of the *commit* event of the model *a0n* or *a0i*. Here, we describe guard discovery.

For the refinement of *a0n*, we therefore first define the function *sigsC*, which reconstructs signal multisets from protocol runs, and use it to obtain the mediator function π^{rs}_{a01} and simulation relation R^{rs}_{a01}, as described in Section 3.4. The multiset $\mathit{sigsC}^{rs}(r)$ contains a Commit signal for each completed responder run. We formalize this as follows.

$$\mathit{sigsC}^{rs}(r)(\mathsf{Commit}([B, \mathsf{S}], (\mathit{Kab}, A, \mathit{Ts}))) \equiv |\mathit{RR}|,$$

where

$$\mathit{RR} \equiv \{\mathit{Rb} \mid \exists \mathit{nl}.\ r(\mathit{Rb}) = (\mathsf{Resp}, A, B, \mathit{Kab}\#\mathit{Ts}\#\mathit{nl})\}.$$

Similarly, completed server runs give rise to $\mathsf{Running}$ signals. Since the session key Kab is derived from the (unique) server run identifier Rs, a simpler definition suffices.

$$\begin{gathered}\mathit{sigsC}^{rs}(r)(\mathsf{Running}([B, \mathsf{S}], (\mathit{Kab}, A, \mathit{Ts}))) \equiv \\ \text{if } \exists \mathit{Rs}, \mathit{Na}, \mathit{nl}.\ (\mathit{Kab} = \mathit{Rs}\$\mathsf{sk} \wedge r(\mathit{Rs}) = (\mathsf{Serv}, A, B, \mathit{Na}\#\mathit{Ts}\#\mathit{nl})) \text{ then } 1 \text{ else } 0.\end{gathered}$$

The existential quantifications on nl account for extensions to the run frames with additional atomic messages in later refinements. Finally, we set $\mathit{sigsC}^{rs}(r)(x) \equiv 0$ at all other points x.

Next, we prove that the server's event $\mathit{step3}_{\mathit{kt1in}}$ and the responder's event $\mathit{step5}_{\mathit{kt1in}}$ refine the events $\mathit{running}_{\mathit{a0n}}([B, \mathsf{S}], (\mathit{Kab}, A, \mathit{Ts}))$ and $\mathit{commit}_{\mathit{a0n}}([B, \mathsf{S}], (\mathit{Kab}, A, \mathit{Ts}))$ of the abstract model *a0n*. The remaining events refine *skip*. In the proof of guard refinement **(GRD)** for $\mathit{step5}_{\mathit{kt1in}}$, we get stuck in a proof state that directly suggests the following authentication guard for this event.

$$B \notin \mathit{bad} \rightarrow \exists \mathit{Rs}, \mathit{Na}, \mathit{nl}.\ (\mathit{Kab} = \mathit{Rs}\$\mathsf{sk} \wedge s.\mathit{runs}(\mathit{Rs}) = (\mathsf{Serv}, A, B, \mathit{Na}\#\mathit{Ts}\#\mathit{nl})). \tag{6}$$

This guard guarantees to an honest B that there is a server in a state counting as a matching $\mathsf{Running}([B, \mathsf{S}], (\mathit{Kab}, A, \mathit{Ts}))$ signal. After adding this guard, the proof succeeds.

For *a0i*'s refinement, we proceed similarly to discover the authentication guard for the event $\mathit{step4}_{\mathit{kt1in}}(\mathit{Ra}, A, B, \mathit{Na}, \mathit{Kab}, \mathit{Ts})$ in the proof that this event refines $\mathit{commit}_{\mathit{a0i}}([A, \mathsf{S}], (\mathit{Kab}, B, \mathit{Na}, \mathit{Ts}))$.

$$A \notin \mathit{bad} \rightarrow \exists \mathit{Rs}, \mathit{nl}.\ (\mathit{Kab} = \mathit{Rs}\$\mathsf{sk} \wedge s.\mathit{runs}(\mathit{Rs}) = (\mathsf{Serv}, A, B, \mathit{Na}\#\mathit{Ts}\#\mathit{nl})).$$

Compared to (6), the absence of the existential quantification on Na reflects that this agreement includes Na.

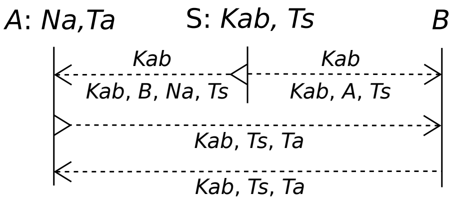


Figure 10: Adding key confirmation (*krb1*).

Proposition 4.2. *$\mathit{kt1in} \sqsubseteq_{\pi^{is}_{a01}} \mathit{a0i}$ for the initiator and $\mathit{kt1in} \sqsubseteq_{\pi^{rs}_{a01}} \mathit{a0n}$ for the responder.*

Finally, it is easy to see that *kt1in* refines *kt1*. The mediator function π_{1in1} removes the nonce *Na* and the timestamp *Ts* from server run frames, and *Ts* from initiator and responder run frames, therefore only keeping the session key *Kab*.

Proposition 4.3. *$\mathit{kt1in} \sqsubseteq_{\pi^{-1}_{\mathit{1in1}},\pi_{\mathit{1in1}}} \mathit{kt1}$.*

4.4.3 Key confirmation

We next extend the model *kt1in* to an abstract model of Kerberos (Figure 10), which achieves key confirmation (**R4**), key freshness for the responder (**R5**), and key recentness (**R6**). To model timestamps and their expiration, we explicitly introduce a (discrete-time) global clock. To keep our model simple, we abstract here from the more realistic scenario where clocks are local and must be synchronized by a separate protocol. For key recentness, the initiator and responder check the validity of a timestamp *Ts* that the server associates with the session key *Kab*. For key confirmation, the initiator and responder mutually agree on the session key *Kab*, its associated timestamp *Ts*, and an initiator timestamp *Ta* (cf. Figure 7). The responder caches keys *Kab* and timestamps *Ta* to obtain an injective agreement with the initiator. We assume arbitrary fixed lifetimes *Ls* and *La* for server and initiator timestamps. Note that the sequence chart in Figure 10 contains all agreements specified in Figure 7 and (partially) orders them causally.

We define the state of *krb1* as the extension of Σ_1 (containing just the *runs* variable, see Section 3.4) with a refined version of the variable *leak* of *kt1* and two additional variables, reflecting the elements discussed above.

$$\begin{aligned}\Sigma_{\mathit{krb1}} \triangleq \Sigma_1 + (\!|\ & \mathit{leak} \in \mathcal{P}(\mathit{key} \times \mathit{agent} \times \mathit{agent} \times \mathit{nonce} \times \mathit{time}),\\ & \mathit{clk} \in \mathit{time}, \mathit{cch} \in \mathcal{P}(\mathit{agent} \times \mathit{key} \times \mathit{time})\ |\!).\end{aligned}$$

Compared to *kt1*, the variable *leak* here associates the complete session context of the server to the leaked key *Kab*, namely the agent names *A* and *B*, the initiator nonce *Na* and the server timestamp *Ts*. This will allow us to prove stronger secrecy guarantees. The variable *clk* models the discrete-time clock. We introduce an associated $\mathit{tick}(T)$ event that increments the clock by T time units. All other events are assumed to take no time and hence do not modify the clock. The variable *cch* represents a cache storing triples $(B, \mathit{Kab}, \mathit{Ta})$ consisting of an agent name B, a session key *Kab*, and an initiator timestamp *Ta*. For replay protection, a responder B checks the cache before accepting a key *Kab* with timestamp *Ta*. A new $\mathit{purge}(B)$ event removes from B's cache those entries whose timestamps *Ta* have expired and thus are no longer valid, which is the case when $s.\mathit{clk} \geq \mathit{Ta} + \mathit{La}$.

The events for Steps 1–5 of the model *krb1* are derived from the corresponding *kt1in* events, possibly adding guards and actions. In Step 3, we associate the timestamp Ts with the current time by adding the guard $Ts = s.clk$.

In the initiator's Step 4, we add the initiator's timestamp Ta as a parameter and record it in the frame, and we introduce two time-related guards. In particular, we have guards that check the validity of timestamps to ensure key recentness (**R6**).

```
$step4_{krb1}(Ra, A, B, Na, Kab, Ts, Ta) \equiv \{(s, s') \mid$       -- by $A$
    $\cdots$                                                        -- guards of $step4_{kt1in}$ (omitted)
    $Ta = s.clk \wedge$                                             -- get timestamp
    $s.clk < Ts + Ls \wedge$                                        -- check validity of $Ts$

    $s'.runs := s.runs(Ra \mapsto (\mathsf{Init}, A, B, [Kab, Ts, Ta]))\ \}.$
```

The first guard states that the timestamp Ta is the current value of the clock. The second guard ensures the validity of the server timestamp Ts.

In the responder's Step 5, we also add Ta as a parameter and record it in the frame. Furthermore, we introduce four new guards and a new action.

```
$step5_{krb1}(Rb, A, B, Kab, Ts, Ta) \equiv \{(s, s') \mid$             -- by $B$
    $\cdots$                                                        -- guards of $step5_{kt1in}$ (omitted)
    -- for agreement with $A$ on $(Kab, Ts, Ta)$
    $(A \notin bad \wedge B \notin bad \rightarrow \exists Ra, nl.\ s.runs(Ra) = (\mathsf{Init}, A, B, Kab\#Ts\#Ta\#nl)) \wedge$

    $(B, Kab, Ta) \notin s.cch \wedge$                              -- replay protection
    $s.clk < Ta + La \wedge$                                        -- check validity of $Ta$
    $s.clk < Ts + Ls \wedge$                                        -- check validity of $Ts$

    $s'.cch := s.cch \cup \{(B, Kab, Ta)\}$                         -- cache update
    $s'.runs := s.runs(Rb \mapsto (\mathsf{Resp}, A, B, [Kab, Ts, Ta]))\ \}.$
```

The first guard ensures agreement with the initiator on the data (Kab, Ts, Ta). The second guard achieves injectivity for the responder by checking that B has not previously seen Kab with timestamp Ta. The last two guards ensure recentness by checking the validity of Ta and Ts. The new action adds (B, Kab, Ta) to the cache to avoid future replays.

We also add a new Step 6 to the initiator, which uses an authentication guard to achieve agreement with a responder run Rb on Kab, Ts, and Ta. We add an arbitrary value END to the frame to mark the initiator run's termination.

```
$step6_{krb1}(Ra, A, B, Na, Kab, Ts, Ta) \equiv \{(s, s') \mid$                 -- by $A$
    $s.runs(Ra) = (\mathsf{Init}, A, B, [Kab, Ts, Ta]) \wedge$

    -- for agreement with $B$ on $(Kab, Ts, Ta)$
    $A \notin bad \wedge B \notin bad \rightarrow (\exists Rb.\ s.runs(Rb) = (\mathsf{Resp}, A, B, [Kab, Ts, Ta])) \wedge$

    $s'.runs := s.runs(Ra \mapsto (\mathsf{Init}, A, B, [Kab, Ts, Ta, END]))\ \}.$
```

Finally, we modify the *leak* event by recording tuples (Kab, A, B, Na, Ts) instead of just Kab and by including the additional guard $s.clk > Ts + Ls$, which reflects the intuition that the lifetime Ls is short enough to prevent the compromise of a session key during its validity period.

The mediator function π_{11} in the refinement of *kt1in* by *krb1* drops the timestamps *Ta* and the termination marker *END* from initiator and responder frames.

Proposition 4.4. *krb1* $\sqsubseteq_{\pi_{11}^{-1},\pi_{11}}$ *kt1in.*

The mediators π^{ir}_{a01} and π^{ri}_{a01} and associated simulation relations for refining *a0i* and *a0n* are defined analogously to Section 4.4.2. The authentication guards can be defined or discovered as described in that section. The replay cache guarantees injective agreement with the initiator to the responder, while the initiator obtains only a non-injective agreement with the responder. The proof of injectivity in the refinement of $commit_{a0i}$ by $step5_{krb1}$ requires an invariant stating that if a responder B knows a key Kab and a timestamp Ta then he has an entry (B, Kab, Ta) in the replay cache during Ta's validity. Appendix B provides some proof details.

Proposition 4.5. *(i) krb1* $\sqsubseteq_{\pi^{ir}_{a01}}$ *a0n for the initiator and (ii) krb1* $\sqsubseteq_{\pi^{ri}_{a01}}$ *a0i for the responder.*

4.4.4 Key freshness

Finally, we formalize key freshness (**R5**) for the responder as an invariant of *krb1*. Recall that key freshness means that there is a unique key for each session. More precisely, this property expresses that a session key K appearing in a run of a responder B with an initiator A uniquely identifies that run, provided that A and B are honest and the session key K has not leaked.

$$\begin{aligned}
&rfresh_{krb1} \equiv \{s \mid \forall R, R', A, A', B, B', K, Ts, Ts', Ta, Ta'.\\
&\quad s.runs(R) = (\mathsf{Resp}, A, B, [K, Ts, Ta]) \wedge\\
&\quad s.runs(R') = (\mathsf{Resp}, A', B', [K, Ts', Ta']) \wedge\\
&\quad B \notin bad \wedge A \notin bad \wedge K \notin dom(s.leak)\\
&\quad \rightarrow R = R'\}.
\end{aligned}$$

We have proved that this is an external invariant of *krb1*.

Proposition 4.6. $oreach(krb1) \subseteq rfresh_{krb1}$.

All cases except for the critical Step 5 by the responder are proved automatically. The proof of Step 5 relies on other invariants proved for *krb1* or inherited from its ancestors. Secrecy and the responder's agreement with the server are used to exclude the cases where the initiator A' or the responder B' in the run R' is dishonest. Otherwise, we use the injective agreement with the initiator to reduce the proof to the initiator's key freshness (proved for *kt1in*).

4.5 Channel protocols (L2)

As described in Section 3.5, at Level 2, we extend the state with a field *chan* for the set of channel messages of type *chmsg*. All fields except *chan* are observable, i.e., the observation function $\pi_{runs,leak,clk,cch}$ projects Σ_{krb2} to Σ_{krb1}.

$$\Sigma_{krb2} \triangleq \Sigma_{krb1} + (\!|\ chan \in \mathcal{P}(chmsg)\ |\!).$$

For the refinement of *krb1*, we use the simulation relation $R_{12} \equiv \Pi_{runs,leak,clk,cch}$ with the identity mediator function. In the protocol events, we add message-receiving guards, which replace the authorization and authentication guards from Level 1 (if any), and actions for

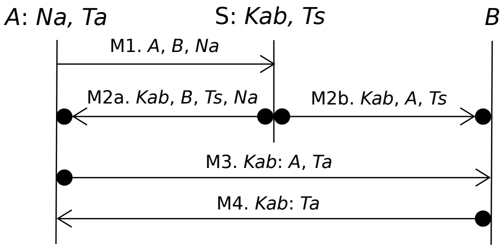


Figure 11: Channel-based Kerberos protocol (*krb2*).

sending channel messages. The local guards remain the same. We refine the leak event to add an insecure message containing the leaked key to the set of channels messages. Moreover, we introduce an active intruder as described in Section 3.5. The intruder's *fake* event refines *skip*, as it only modifies *chan*, while all other events refine their counterparts in the model *krb1*.

The channel-based refinement *krb2* of *krb1* is shown in Figure 11. The protocol is now started by the initiator sending A, B together with his nonce Na to the server. The server uses static secure channels to send the session key Kab, the name B, the timestamp Ts, and the nonce Na to the initiator A and Kab, A, Ts to the responder B. The responder B obtains key confirmation from A by receiving A, Ta on a dynamic authentic channel protected by the session key Kab (and similarly for A's guarantee in the other direction). No confidentiality is required in the key confirmation phase.

As example events, we describe the changes in Steps 3 and 5. In the server's Step 3, we add an additional guard for receiving message M1 and an action for sending messages M2a and M2b.

$$\begin{aligned} &\mathsf{Insec}([A, B, Na]) \in s.chan, \qquad \texttt{-- receive M1} \\ s'.chan := s.chan \cup \{&\mathsf{Secure}(\mathsf{S}, A, [Kab, B, Ts, Na]), \mathsf{Secure}(\mathsf{S}, B, [Kab, A, Ts])\}. \end{aligned}$$

In Step 5, the responder B receives message M2b from the server and M3 from A and sends message M4. Here, the message-receiving guards replace the previous authorization and authentication guards.

$$\begin{aligned} &\mathsf{Secure}(\mathsf{S}, B, [Kab, A, Ts]) \in s.chan, \\ &\quad \mathsf{dAuth}(Kab, [A, Ta]) \in s.chan, \\ s'.chan &:= s.chan \cup \{\mathsf{dAuth}(Kab, [Ta])\}. \end{aligned}$$

We can now prove several invariants related to session key secrecy and compromise. The following invariant expresses that the addition of a set of keys KS to the intruder's initial knowledge does not reveal any additional keys.[1] We have proved a similar invariant for nonces.

$$sesK_{krb2} \equiv \{s \mid \forall KS, K.\ K \in extr_{KS \cup ik_0}(s.chan) \leftrightarrow K \in KS \vee K \in extr_{ik_0}(s.chan)\}.$$

A group of three invariants express the session key's secrecy from each role's point of view. More precisely, these invariants state that the intruder can extract a session key only if it has

[1] This is the L2 equivalent of the session key compromise invariants in [59].

leaked from a session context matching the role's context (i.e., its frame and any fresh values it might have generated). In the case of the initiator, this invariants reads as follows.

$$\begin{aligned}&\mathit{ikk\_init}_{krb2} \equiv \{s \mid \forall Ra, A, B, Kab, Ts, nl.\\ &\quad s.runs(Ra) = (\mathsf{Init}, A, B, Kab\#Ts\#nl) \wedge\\ &\quad Kab \in extr_{ik_0}(s.chan) \wedge A \in good \wedge B \in good \rightarrow\\ &\quad (Kab, A, B, Ra\$\mathsf{na}, Ts) \in s.leak\}.\end{aligned}$$

This strengthens the guarantee resulting from the refinement of the secrecy model $s0$ by taking into account the role's local view and its complete context. By combining the conclusion with an auxiliary invariant stating that $Kab \in dom(s.leak)$ implies $s.clk \geq Ts + Ls$, we know that the intruder can extract at most expired session keys. The server and responder's version of this invariant are similar, but the nonce in the responder's conclusion is existentially quantified, since it is not part of its context. Of these three invariants, only the server's version is needed in the refinement proof.

The refinement proof also requires invariants related to authentication. These are directly suggested by the guard strengthening proof obligations stating that the message-receiving guards at Level 2 imply the security guards at Level 1 (cf. Example 3.8). This is one of the main benefits of using guard protocols as a link between the properties and the message-based protocols.

We can now state the following refinement result. Appendix C contains a proof sketch of guard strengthening for Step 5.

Proposition 4.7. *Let I_{krb2} be the intersection of the invariants of krb2 and let $R'_{12} \equiv R_{12} \cap (\Sigma_{krb1} \times I_{krb2})$. Then (i) $reach(krb2) \subseteq I_{krb2}$ and (ii) $krb2 \sqsubseteq_{R'_{12},id} krb1$.*

4.6 Cryptographic protocols (L3)

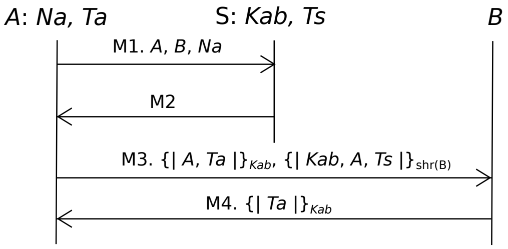
As described in Section 3.6, in our setup at Level 3, each agent A shares a long-term symmetric key $\mathsf{shr}(A)$ with the server S. We concretize the initial key setup relation by defining $keySetup \equiv \{(\mathsf{shr}(A), C) \mid C = A \vee C = \mathsf{S}\}$, thereby establishing Assumption **A3**. In the state, we replace the set of channel messages, *chan*, by a set of cryptographic messages, *IK* (for intruder knowledge). All fields except *IK* are observable. Initially, *IK* holds the corrupted long-term keys, i.e., $\mathsf{shr}(bad)$, which models Assumption **A2**.

$$\Sigma_{krb3v} \triangleq \Sigma_{krb1} + (\!|\ IK \in \mathcal{P}(msg)\ |\!).$$

The simulation relation R_{23} with *krb2* is defined as the intersection of the relations R_{23}^{msgs}, R_{23}^{key}, and R_{23}^{non} with $\Pi_{runs,leak,clk,cch}$. In order to take session key compromise into account, the relation R_{23}^{key} generalizes the basic version from Section 3.6 as follows.

$$\begin{gathered}R_{23}^{key} \equiv \{(s,t) \mid \forall KS, K.\\ KS \subseteq ran(\mathsf{sesK}) \rightarrow (K \in analz(KS \cup t.IK) \leftrightarrow K \in extr_{KS \cup ik_0}(s.chan))\}.\end{gathered}$$

We generalize the relation R_{23}^{non} analogously. The relation R_{23}^{msgs} is parametrized by the protocol-dependent message abstraction function $absMsg \in \mathcal{P}(msg) \rightarrow \mathcal{P}(chmsg)$, which we will instantiate to the Kerberos protocols below.



krb3iv: M2 = $\{\!|\, Kab, B, Ts, Na, \{\!|\, Kab, A, Ts \,|\!\}_{\mathsf{shr(B)}} \,|\!\}_{\mathsf{shr(A)}}$

krb3v: M2 = $\{\!|\, Kab, B, Ts, Na \,|\!\}_{\mathsf{shr(A)}}, \{\!|\, Kab, A, Ts \,|\!\}_{\mathsf{shr(B)}}$

Figure 12: Cryptographic core Kerberos protocols.

By refining the channel-based intruder into a standard Dolev-Yao intruder, as described in Section 3.6, we also establish Assumption **A1**. Moreover, the leak event now adds leaked session keys to the intruder knowledge IK, thus realizing Assumption **A4**. In the protocol events, we replace the channel messages by cryptographic ones. In general, there are alternative realizations using different cryptographic operations.

Figure 12 shows the core of Kerberos 4 [66] and Kerberos 5 [55]. We implement the static secure channels by encryption with the long-term keys and we refine the dynamic authentic channels into encryptions with session keys. (In the Dolev-Yao model, symmetric encryption also provides authenticity.)

We also modify the communication topology of the channel-based model: The initiator now relays the responder's ticket from the server. While in Kerberos 5 the ticket is sent alongside the ciphertext containing the initiator's session key, it appears inside the ciphertext in Kerberos 4. We now present our models of Kerberos 5 and Kerberos 4, *krb3v* and *krb3iv*.

4.6.1 Kerberos 5 protocol (L3)

We focus on the refinement of Steps 3–5, which reflect the modified communication topology. In Step 3, the server sends the message M2 by adding it to IK. This message consists of a pair of ciphertexts and replaces the messages M2a and M2b in *krb2*. In Step 4, the initiator A receives M2 and forwards its second component along with the *authenticator* $\{\!|A, Ta|\!\}_{Kab}$, which proves that A knows Kab.

$$\langle \{\!|Kab, B, Ts, Na|\!\}_{\mathsf{shr}(A)}, X\rangle \in s.IK \qquad \texttt{-- recv M2}$$

$$s'.IK := s.IK \cup \{\langle \{\!|A, Ta|\!\}_{Kab}, X\rangle\}. \qquad \texttt{-- send M3}$$

The responder receives the two-component message M3 in Step 5 and sends back the confirmation message M4.

$$\langle \{\!|A, Ta|\!\}_{Kab}, \{\!|Kab, A, Ts|\!\}_{\mathsf{shr}(B)}\rangle \in s.IK \qquad \texttt{-- recv M3}$$

$$s'.IK := s.IK \cup \{\{\!|Ta|\!\}_{Kab}\}. \qquad \texttt{-- send M4}$$

The message abstraction function, *absMsg*, abstracts the components of messages M2 and M3 separately. For instance, here are the rules defining the abstraction of the initiator's encrypted key, the responder tickets, and the authenticators.

$$\frac{\{\!|K, B, T, N|\!\}_{\mathsf{shr}(A)} \in H}{\mathsf{Secure}(\mathsf{S}, A, [K, B, T, N]) \in \mathit{absMsg}(H)} \qquad \frac{\{\!|K, A, T|\!\}_{\mathsf{shr}(B)} \in H}{\mathsf{Secure}(\mathsf{S}, B, [K, A, T]) \in \mathit{absMsg}(H)}$$

$$\frac{\{\!|A, T|\!\}_{K} \in H}{\mathsf{dAuth}(K, [A, T]) \in \mathit{absMsg}(H)}.$$

The possibility to abstract message parts, which is reflected in the definition of the simulation relation as $\mathit{absMsg}(\mathit{parts}(s.\mathit{IK})) \subseteq t.\mathit{chan}$ (cf. Section 3.6.3), allows us to abstract each component of message M2 separately. This enables the modification of the communication topology between Levels 2 and 3.

The refinement proof requires only two additional invariants. The first one, called $\mathit{ltK}_{\mathit{krb3v}}$, states that the long-term keys the intruder knows are exactly those of the dishonest agents (Assumption **A2**). The second invariant is the L3-equivalent of invariant $\mathit{sesK}_{\mathit{krb2}}$, which we can however directly derive from that invariant and the simulation relation. We have already established all other relevant properties on higher levels of abstraction.

Proposition 4.8. *Let* $R^{v}_{23} \equiv R_{23} \cap (\mathit{sesK}_{\mathit{krb2}} \times \mathit{ltK}_{\mathit{krb3v}})$. *Then we have* $\mathit{reach}(\mathit{krb3v}) \subseteq \mathit{ltK}_{\mathit{krb3v}}$ *and* $\mathit{krb3v} \sqsubseteq_{R^{v}_{23},\mathit{id}} \mathit{krb2}$.

4.6.2 The Kerberos 4 protocol (L3)

In the core Kerberos 4 protocol, the responder's ticket is encrypted inside the initiator's message from the server. Hence, message M2 is modified as follows.

M2'. $\mathsf{S} \to A : \{\!|\mathit{Kab}, B, \mathit{Ts}, \mathit{Na}, \{\!|\mathit{Kab}, A, \mathit{Ts}|\!\}_{\mathsf{shr}(B)}|\!\}_{\mathsf{shr}(A)}$.

The changes in the model are straightforward. Moreover, we must slightly adapt the simulation relation as follows. In the message abstraction function, the cryptographic messages abstracted to message M2a, i.e., $\mathsf{Secure}(\mathsf{S}, A, [K, B, T, N])$ (see Figure 11), are now of the form $\{\!|K, B, T, N, X|\!\}_{\mathsf{shr}(A)}$ and include a message variable X for the responder's ticket instead of $\{\!|K, B, T, N|\!\}_{\mathsf{shr}(A)}$ for Kerberos 5. Moreover, we have to weaken the equivalences in the relations R^{key}_{23} and R^{non}_{23} to implications, since the double encryption of the responder's key has no correspondent on L2. As a consequence, we can no longer derive the session key compromise invariants from the L2 versions and the simulation relation. Instead, we must reprove them at this level.

The refinement proof for *krb3iv* requires an additional invariant, which links the variable X above with the ticket by describing the ticket's shape and the encrypted key K in message M2 received by an honest initiator A.

$$\begin{aligned}&\mathit{ticket}_{\mathit{krb3iv}} \equiv \{s \mid \forall A, B, T, N, K, X.\\ &\quad \{\!|K, B, T, N, X|\!\}_{\mathsf{shr}(A)} \in \mathit{parts}(s.\mathit{IK}) \land A \notin \mathit{bad} \to X = \{\!|K, A, T|\!\}_{\mathsf{shr}(B)} \land K \in \mathit{ran}(\mathsf{sesK})\}.\end{aligned}$$

We then prove:

Proposition 4.9. *Let* I_{krb3iv} *be the intersection of the invariants of krb3iv and let* $R^{iv}_{23} \equiv R'_{23} \cap (\mathit{sesK}_{\mathit{krb2}} \times I_{\mathit{krb3iv}})$. *Then* $\mathit{reach}(\mathit{krb3iv}) \subseteq I_{\mathit{krb3iv}}$ *and* $\mathit{krb3iv} \sqsubseteq_{R^{iv}_{23},\mathit{id}} \mathit{krb2}$.

4.7 Discussion

4.7.1 Overall security guarantees at Level 3

Having gone through a number of refinements, the reader may wonder at this point what is the precise relationship between the secrecy and authentication properties proved as invariants at Level 0 and the final models at Level 3. The answer is given by Proposition 2.6 and Corollary 2.8, which state that refinement is transitive and that *external* invariants (such as the secrecy and agreement invariants of *s0*, *a0n*, and *a0i*) are preserved along refinements. The mediator function translates the observations from concrete to abstract models.

As an example, consider the series of refinements in Figure 6: we have refined the secrecy model *s0* in five steps into the core Kerberos 5 model (*krb3v*). The composed mediator function along the right-hand side of this figure is $\pi_{\mathit{s01}} \circ \pi_{\mathit{1in1}} \circ \pi_{11}$. The first part, $\pi_{\mathit{1in1}} \circ \pi_{11}$, projects the state of the model *krb3v* with fields *runs*, *clk*, *cch*, and *IK* to the state of the model *kt1*, which has only the *runs* variable. Moreover, using the notation of Figure 6, the frames of the initiator, responder, and server runs are projected from $([\mathit{Kab}, \mathit{Ts}, \mathit{Ta}], [\mathit{Kab}, \mathit{Ts}, \mathit{Ta}, \mathit{END}], [\mathit{Na}, \mathit{Ts}])$ to $([\mathit{Kab}], [\mathit{Kab}], [])$, thereby removing everything but the session key *Kab*.

The second part, the mediator function π_{s01} (defined in Section 4.4.1), transforms this minimal state information to the knowledge and authorization relations *kn* and *az* of the model *s0*. Hence, using Proposition 2.4, the following overall secrecy result can be derived as a combination of Propositions 3.1, 4.1, 4.3, 4.4, 4.7, and 4.8.

Corollary 4.10. $(\pi_{\mathit{s01}} \circ \pi_{\mathit{1in1}} \circ \pi_{11})(\mathit{oreach}(\mathit{krb3v})) \subseteq \mathit{secrecy}$.

In a similar way, we can express the authentication results directly as properties of *krb3v* (cf. Figure 7). The initiator's injective agreement with the server on $(\mathit{Kab}, B, \mathit{Na}, \mathit{Ts})$ and his non-injective agreement on $(\mathit{Kab}, \mathit{Ts}, \mathit{Ta})$ with the responder are summarized in the following corollary.

Corollary 4.11. *For the initiator, we have*

1. $(\pi^{is}_{\mathit{a01}} \circ \pi_{\mathit{1in1}} \circ \pi_{11})(\mathit{oreach}(\mathit{krb3v})) \subseteq \mathit{iagree}$*, and*

2. $(\pi^{ir}_{\mathit{a01}} \circ \pi_{11})(\mathit{oreach}(\mathit{krb3v})) \subseteq \mathit{niagree}$*.*

We complete the picture by stating the authentication guarantees for the responder with the server and with the initiator.

Corollary 4.12. *For the responder, we have*

1. $(\pi^{rs}_{\mathit{a01}} \circ \pi_{\mathit{1in1}} \circ \pi_{11})(\mathit{oreach}(\mathit{krb3v})) \subseteq \mathit{niagree}$*, and*

2. $(\pi^{ri}_{\mathit{a01}} \circ \pi_{11})(\mathit{oreach}(\mathit{krb3v})) \subseteq \mathit{iagree}$*.*

Summarizing, given a security property P proved as an external invariant of a model S (e.g., at Level 0) and a series of refinements of S into a model S' (e.g., at Level 3) with composed mediator function π, Proposition 2.4 yields the guarantee $\pi(\mathit{oreach}(S')) \subseteq P$ for S', i.e., π transforms the set of concrete observations of S' to a set of abstract observations satisfying P.

Table 4: Overall specification and proof statistics

	Definitions	Lemmas	Lines	CPU time
Infrastructure	57	327	3421	25 sec
Level 1 models	50	146	2107	46 sec
Kerberos 5	83	166	3055	3 min 44 sec
NSSK	80	192	3234	2 min 41 sec
Denning-Sacco	60	89	1857	1 min 02 sec

Table 5: Detailed proof statistics: lemmas, inductive/derived/inherited invariants, and refinements

Level	Model	Lemmas	Invariants	Refines
L1	*kt1*	31	1 / 0 / 1	*s0*
	kt1in	62	2 / 0 / 3	*a0i*, *a0n*
	kt1nn	53	1 / 0 / 0	*a0n* (twice)
	krb1	81	3 / 0 / 5	*kt1in*, *a0i*, *a0n*
	nssk1	74	2 / 0 / 5	*kt1in*, *a0n* (twice)
L2	*krb2*	57	9 / 3 / 2	*krb1*
	nssk2	84	12 / 3 / 2	*nssk1*
L3	*krb3iv*	40	4 / 0 / 0	*krb2*
	krb3v	28	1 / 0 / 1	*krb2*
	nssk3	34	3 / 0 / 0	*nssk2*

4.7.2 Specification and proof statistics

We summarize some statistics from our developments in Table 4, which has entries for five groups of theories. For each group, we indicate the number of definitions and lemmas that we formalized in Isabelle/HOL, the number of lines of the corresponding theory files, and the CPU time. The times are for proof checking only and do not include the generation of a session image or the documentation. The measurements were made on a 2.6 GHz Intel Core i7 laptop with 8 GB RAM running Isabelle/HOL 2016-1.

The first group consists of 11 infrastructure theories, which support our method, and includes our general theory of refinement (Section 2.2) and our infrastructure for security protocol modeling and refinement including the L0 models of secrecy and authentication (Section 3). These theories can be reused in other developments. The second group consists of the L1 models *kt1*, *kt1in*, and *kt1nn* (Sections 4.4.1 and 4.4.2). The third, fourth, and fifth lines list the data for the models *krb1*, *krb2*, and *krb3v* pertaining to the Kerberos 5 protocol (Sections 4.4.3, 4.5, and 4.6.1), the models *nssk1*, *nssk2*, and *nssk3* related to the Needham-Schroeder Shared-Key (NSSK) protocol, and the models *ds1*, *ds2*, and *ds3* for the Denning-Sacco protocol (see also Figure 5).

Table 5 shows more detailed proof statistics for the models used in our development of the Kerberos 4 and 5 protocols and the NSSK protocol. In particular, the fourth column lists three numbers for invariants. The first number denotes *inductive* invariants, which are proved by induction and primarily used to strengthen the simulation relation in refinement proofs. One type of (internal) invariant, which we find in many models at Levels 1 and 2, are key definedness invariants. These state that session keys are only generated by existing runs, i.e., the run R of a used session keys $R\$sk$ appears in the domain of the variable *runs*; they are easy to prove and serve mainly a technical purpose. These invariants are therefore not further mentioned below. The second number denotes *derived* invariants, which follow from other invariants of the same model. The third number indicates invariants that are *inherited* from higher-level models. This number does not include all inherited invariants, but only those that are needed in a proof at the same or a lower level. For example, the abstract Kerberos and NSSK models, *krb1* and *nssk1*, inherit the same five invariants from their common L0 and L1 ancestor models. Hence, these invariants can be used in further proofs without the need to reestablish them.

As mentioned earlier, all system requirements are already established at Level 1, mainly in the form of refinements of L0 models. Exceptions are key freshness properties, which are formulated as invariants. There is one other new invariant at Level 1, which states a property of the cache in the abstract Kerberos model *krb1*.

The largest number of invariants is required at Level 2. These invariants can be classified into two groups: those concerning session key secrecy or compromise (6 each for *krb2* and *nssk2*) and those relating received messages to their sender's or receiver's state (5 for *krb2* and 6 for *nssk2*). Many of these invariants arise naturally from guard refinement proof obligations, for example, to establish that the received messages imply an authorization or authentication guard at Level 1. Most of them have short straightforward proofs (2–6 lines of proof script).

The invariants remaining at Level 3 mainly concern details introduced at that level. Examples are the secrecy of long-term keys, an invariant about the shape of forwarded tickets that are encrypted with an honest agent's long-term key (*krb3iv* and *nssk3*), and an invariant expressing that session keys are not used to encrypt other session keys or nonces (*krb3iv* and *nssk3*).

5 Related work

There have been other proposals for developing security protocols by refinement using various formalisms such as the B method [17], its combination with CSP [24], Event-B [16], I/O automata [43], and ASMs [15]. None of these continue their refinements to the level of a full Dolev-Yao intruder. Either they only consider an intruder that is passive [43], defined ad-hoc [24, 15, 16], or that corresponds to our L2 intruder [17]. This makes a comparison of their results with standard protocol models difficult. Moreover, these works do not propose a uniform and systematic development method as we do with our four-level refinement strategy and most of them develop individual protocols rather than entire families.

Several authors have studied protocol transformations within the cryptographic level, L3. Hui and Lowe [35] define a family of syntactic transformations and prove their soundness. Nguyen and Sprenger [56, 57] extend this work to the untyped case and to equational theories. The focus of these works is on improving the performance of automatic verification tools rather than on protocol development. Guttman [34] studies a rich class of protocol transformations in the strand space model and proves their soundness. His approach is based on the simulation of protocol analysis steps instead of execution steps. Each such analysis step explains the origin of a message. Datta et al. [30] prove properties of protocol classes specified using messages containing function variables. Refinement here means instantiating function variables and discharging the associated assumptions. Pavlovic et al. [60, 26] propose a logical approach to proving such refinements. However, the soundness of these refinements is not formally justified. In contrast to our approach, the transformations discussed in this paragraph do not involve a fundamental change of the abstraction level as they all work with cryptographic messages (Level 3).

Abstract channels with security properties and their transformations were studied by Maurer and Schmid [46]. Boyd has formalized analogous results using Z [20]. Bieber et al. [18] model abstract channels using the B method and refine them to cryptographic implementations. Abadi et al. [1] formalize secure channels in a process calculus and establish full abstraction results for translations to cryptographic implementations. Several works have investigated the use of abstract channels to model and verify layered protocols, e.g., an application protocol running on top of a protocol establishing a secure channel (such as TLS). Bella et al. [12] explore the modeling of such layered protocols based on channel abstractions in Paulson's inductive approach [59]. Kamil and Lowe [38] prove the soundness of such channel abstractions under certain independence conditions and show in [39] that TLS satisfies these conditions. Mödersheim et al. [52, 33, 53] propose a compositional approach, where the protocols implementing the channel and the (possibly cryptographic) application protocol can be verified separately. Their objective is to provide composition theorems guaranteeing that, under certain disjointness conditions, security properties are preserved by layered composition. In contrast, our focus in on refining channel protocols into concrete cryptographic protocols.

Classical notions of refinement (such as simulation) do not preserve information-flow properties, since they involve a reduction of non-determinism, which can destroy secrecy. Several works address this problem, known as the refinement paradox, for example, [45, 5, 37, 47]. Morgan and McIver [47, 50, 51] solve the paradox by explicitly recording the set of possible values of secret variables. These sets represent the intruder's ignorance and refinements may extend, but never reduce them. Cortier et al. [28] show that strong secrecy is equivalent to reachability-based secrecy (used here), if the secrets are not tested. Key establishment protocols that include a key confirmation phase (such as such as Kerberos and NSSK) do not satisfy this condition.

Simulation-based security [25, 7] is a paradigm for specifying idealized functionalities and implementing them using a notion of secure emulation. Delaune et al. [31] have proposed a symbolic version of this paradigm, which can be understood as a form of compositional refinement. The compositionality comes at the price of requiring an public-key encryption/decryption functionality and proving a joint-state theorem. As an example, they derive the Needham-Schroeder-Lowe protocol. The modeling of encryption as a service considerably reduces the abstraction level of these models compared to the standard symbolic representation as a term constructor.

Our most concrete models are still quite abstract when compared to a protocol implementation in a programming language such as C or Java. The work by Polikarpova and Moskal [61] can be seen as an extension of our refinement levels with two additional levels leading towards an implementation: one in which the messages are replaced by bitstrings and one that represents the real implementation. The refinements are encoded and proved in the general-purpose C program verifier VCC.

Isabelle/HOL has been used in several approaches to post-hoc security protocol verification. Paulson [59] uses induction to define the protocols' event traces and verify their properties. We reuse his Isabelle/HOL theory of cryptographic messages including the closure operators *parts*, *analz*, and *synth* in our L3 refinements. Refinement enables us to prove most security properties at higher levels of abstraction. Moreover, strong authentication properties such as injective agreement cannot be proved in his models, since any message may trigger the same response multiple times. More recently, Isabelle/HOL has been used to machine-check proofs that are generated by automatic security protocol verifiers [21, 48].

Several other researchers have analyzed Kerberos. Bella and Riccobene [15] develop Kerberos 4 in three refinements using ASMs. They use a non-standard attacker model and prove mostly liveness properties (for example, all runs reach a specific state) instead of secrecy and authentication properties. Bella and Paulson model BAN Kerberos [14], Kerberos 4 [13], and Kerberos 5 [11] including session key compromise using the inductive approach [59]. However, they do not model a replay cache and prove only non-injective agreements. Butler et al. [23, 22] constructed detailed models of Kerberos 5 using multiset rewriting. These models include cross-realm authentication and other realistic features such as options, flags, and error handling. They manually construct their models and proofs in several "refinements" to keep them manageable. However, their notion of refinement is informal.

6 Conclusions

Our development provides strong evidence that refinement supports the systematic understanding and development of families of protocols. The abstract models help the developer to focus on the essentials: In our case studies, we have established all requirements on guard protocol models (L1), which contain neither messages, communication channels, nor intruder events. Our refinement strategy guides the developer towards the concrete levels that account for the environment assumptions, namely, the distributed environment controlled by a Dolev-Yao intruder. The abstraction levels of our refinement strategy are reflected in well-structured proofs of correctness, where the simulation relations used are either fixed (a projection at L1–L2) or systematically derived (for example, abstraction of runs to signals at L0–L1 and cryptographic to channel messages at L2–L3). Our case studies also show that our development strategy and tools scale to realistic protocols with non-trivial features. Our approach may also help in the standardization of security protocols [9, 10]. Standards frequently include several variants of a protocol, for example, designed for different cryptographic setups or using different forms of key establishment (e.g., key transport versus Diffie-Hellman key agreement). Here, one could use our method to identify an abstract protocol model capturing the commonalities of the different variants and prove their properties once and for all. The different concretizations can then be obtained by refinements.

A central part of this work has been the development and exploitation of guard protocols, which form the bridge between security properties and channel protocols, i.e., from the "what" to the "how". Security guards realize properties abstractly. Moreover, they substantially simplify proof construction. They give rise to invariants in a canonical way during refinement, thereby facilitating invariant discovery. These invariants strengthen the simulation relations in the refinement proofs.

Our channel protocol model is quite simple and protocol messages with nested cryptographic operations or undecryptable message parts have no direct representation. This excludes modeling, for example, messages containing certificates, the forwarding of undecryptable messages, and nested encryption (NSSK and Kerberos). Our experience has convinced us that this simplicity is a virtue rather than a limitation. Our models of server-based key transport protocols naturally reflects their actual (star-shaped) security architecture. We view forwarding and double encryption as implementation techniques, to be dealt with at the final level. Our developments show that this is possible. Such abstractions are even more beneficial for developing new protocols. From this perspective, certificates provide an abstract authentic channel from the certification authority to the agent verifying the certificate's content and encrypted and signed messages are just one way of implementing a secure channel.

Future work We ultimately envision a tool-based development process where engineers can choose standard properties and follow high-level recipes for building guard, channel, and crypto protocols, with tools checking their steps along the way. To achieve this, we will work into two directions. First, we want to extend the range of protocols that can be modeled and reasoned about. For example, we plan to add support for Diffie-Hellman key agreement, compromising adversaries, and more complex properties such as perfect forward secrecy, possibly along the lines of [8]. Recent work in this direction is [40], which in addition to session key compromise also covers different forms of dynamic agent compromise. Second, we would like to automate development based on our strategy. It should be possible to derive protocol models directly from high-level descriptions such as the authentication graphs of Figure 7 and sequence charts of Figures 8–10. Moreover, with suitable infrastructure it should be feasible to automatically generate and (as far as possible) prove invariants and simulations, given their strong regularity.

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A Definitions of *synth*, *analz*, and *parts*

For completeness, we include the definitions of the closure operators *synth*, *analz*, and *parts* on cryptographic messages from [59]. The rules for synthesizing messages are given in Figure 13. The first rule injects the set of messages H into $\mathit{synth}(H)$. The next two rules model that agent names and timestamps are public values. The final two rules enable the attacker to construct pairs, ciphertexts, and signatures.

$$\frac{M \in H}{M \in \mathit{synth}(H)} \qquad \frac{A \in \mathit{agent}}{A \in \mathit{synth}(H)} \qquad \frac{T \in \mathbb{N}}{T \in \mathit{synth}(H)}$$

$$\frac{M_1 \in \mathit{synth}(H) \qquad M_2 \in \mathit{synth}(H)}{\langle M_1, M_2 \rangle \in \mathit{synth}(H)} \qquad \frac{M \in \mathit{synth}(H) \quad K \in \mathit{synth}(H)}{\{\!|M|\!\}_K \in \mathit{synth}(H)}$$

Figure 13: The rules defining *synth*.

$$\frac{M \in H}{M \in \mathit{analz}(H)} \qquad \frac{\langle M_1, M_2 \rangle \in \mathit{analz}(H)}{M_i \in \mathit{analz}(H)} \qquad \frac{\{\!|M|\!\}_K \in \mathit{analz}(H) \quad K^{-1} \in \mathit{analz}(H)}{M \in \mathit{analz}(H)}$$

$$\frac{M \in H}{M \in \mathit{parts}(H)} \qquad \frac{\langle M_1, M_2 \rangle \in \mathit{parts}(H)}{M_i \in \mathit{parts}(H)} \qquad \frac{\{\!|M|\!\}_K \in \mathit{parts}(H)}{M \in \mathit{parts}(H)}$$

Figure 14: The rules defining *analz* and *parts*.

The rules in Figure 14 define the decomposing closure operators *analz* and *parts*. The set $\mathit{analz}(H)$ represents the set of messages that an attacker can extract from the set H using the available cryptographic keys, whereas the set $\mathit{parts}(H)$ represents all messages that the attacker could extract from H if he knew all keys. The first rule in each row injects the set H into the closure. The second rule in each row models the projection of pairs on each component ($i \in \{1, 2\}$). The final rules enable the extraction of cleartexts from ciphertexts and signatures. For *analz*, the extraction of M from $\{\!|M|\!\}_K$ requires the inverse key K^{-1}, defined by $\mathsf{pub}(A)^{-1} = \mathsf{pri}(A)$, $\mathsf{pri}(A)^{-1} = \mathsf{pub}(A)^{-1}$, and $K^{-1} = K$ for symmetric keys K.

B Proof of Step 5 in Proposition 4.5(ii) (L1)

We sketch the proof of guard strengthening that arises in the refinement of the abstract event $\mathit{commit}_{a0i}([B, A], (\mathit{Kab}, \mathit{Ts}, \mathit{Ta}))$ by the concrete $\mathit{step5}_{\mathit{krb1}}(\mathit{Rb}, A, B, \mathit{Kab}, \mathit{Ts}, \mathit{Ta})$ of the responder B, which is part of establishing the responder's injective agreement with the initiator. We define the function $\mathit{sigsC}(r)$ that reconstructs the abstract signals from the runs map r as follows.

$$\mathit{sigsC}(r)(\mathsf{Running}([B, A], (\mathit{Kab}, \mathit{Ts}, \mathit{Ta}))) \equiv c_I(r),$$
$$\mathit{sigsC}(r)(\mathsf{Commit}([B, A], (\mathit{Kab}, \mathit{Ts}, \mathit{Ta}))) \equiv c_R(r).$$

Here, $c_I(r)$ and $c_R(r)$ are the following cardinalities.

$$c_I(r) \equiv \big|\{\mathit{Ra} \mid \exists nl.\, r(\mathit{Ra}) = (\mathsf{Init}, A, B, \mathit{Kab}\#\mathit{Ts}\#\mathit{Ta}\#nl)\}\big|,$$
$$c_R(r) \equiv \big|\{\mathit{Rb} \mid \exists nl.\, r(\mathit{Rb}) = (\mathsf{Resp}, A, B, \mathit{Kab}\#\mathit{Ts}\#\mathit{Ta}\#nl)\}\big|.$$

For the remaining cases, we set $\mathit{sigsC}(r)(x) \equiv 0$. Guard strengthening requires that we prove the assertion $c_R(r) < c_I(r)$, assuming that A and B are honest and that the guards of $\mathit{step5}_{\mathit{krb1}}$ are satisfied. Since we use a cache to prevent a responder B from accepting the same key Kab and timestamp Ta multiple times, there cannot be any prior execution of Step 5 with these parameters. We therefore strengthen the subgoal $c_R(r) < c_I(r)$ to the conjunction of the following two subgoals: $c_I(r) > 0$ and $c_R(r) = 0$. The first subgoal follows from the authentication guard for $\mathit{step5}_{\mathit{krb1}}$. The second subgoal requires that there is no responder run Rb' and list nl' corresponding to a commit signal, i.e.,

$$s.\mathit{runs}(\mathit{Rb}') = (\mathsf{Resp}, A, B, \mathit{Kab}\#\mathit{Ts}\#\mathit{Ta}\#nl'). \tag{7}$$

We prove this by contradiction, using an invariant stating that (7) and the validity of Ta (i.e., $s.\mathit{clk} < \mathit{Ta} + \mathit{La}$) entail the existence of a cache entry $(B, \mathit{Kab}, \mathit{Ta}) \in s.\mathit{cch}$. This entry was added in a previous execution of Step 5 by run Rb' and is not yet purged from the cache, since the timestamp Ta is still valid. Hence, by assuming (7) and noting that $s.\mathit{clk} < \mathit{Ta} + \mathit{La}$ and the replay check $(B, \mathit{Kab}, \mathit{Ta}) \notin s.\mathit{cch}$ are guards of Step 5, we use the invariant to derive a contradiction. This concludes the proof that the guards of the $\mathit{step5}_{\mathit{krb1}}$ event imply the guard of commit_{a0i}.

C Proof of Step 5 in Proposition 4.7(ii) (L2)

The guard strengthening proof obligation for the responder's $\mathit{step5}_{\mathit{krb2}}$ requires that the guards for receiving M2b and M3 and for the timestamps' validity imply the three security guards of $\mathit{step5}_{\mathit{krb1}}$: the authorization guard (8), the server authentication guard (9), and initiator authentication guard (10):

$$\mathit{Kab} \notin \mathit{dom}(s.\mathit{leak}) \rightarrow (\mathit{Kab}, B) \in \mathit{azC}(s.\mathit{runs}), \tag{8}$$
$$B \notin \mathit{bad} \rightarrow \exists \mathit{Rs}, \mathit{Na}.\, \mathit{Kab} = \mathit{Rs}\$\mathsf{sk} \wedge s.\mathit{runs}(\mathit{Rs}) = (\mathsf{Serv}, A, B, [\mathit{Na}, \mathit{Ts}]), \tag{9}$$
$$A \notin \mathit{bad} \wedge B \notin \mathit{bad} \rightarrow \exists \mathit{Ra}, nl.\, s.\mathit{runs}(\mathit{Ra}) = (\mathsf{Init}, A, B, \mathit{Kab}\#\mathit{Ts}\#\mathit{Ta}\#nl). \tag{10}$$

The following invariant directly arises from the proof obligation expressing the strengthening of the server authentication guard (9). It expresses the guarantee that an honest B gets about the server's state from receiving message M2b.

$$\begin{array}{l} \mathit{M2}_{\mathit{krb2}} \equiv \{s \mid \forall \mathit{Kab}, A, B, \mathit{Ts}. \\ \quad \mathsf{Secure}(\mathsf{S}, B, [\mathit{Kab}, A, \mathit{Ts}]) \in s.\mathit{chan} \land B \notin \mathit{bad} \rightarrow \\ \qquad \exists \mathit{Rs}, \mathit{Na}.\ \mathit{Kab} = \mathit{Rs}\$\mathsf{sk} \land s.\mathit{runs}(\mathit{Rs}) = (\mathsf{Serv}, A, B, [\mathit{Na}, \mathit{Ts}])\}. \end{array}$$

An additional invariant describes the reasons for the intruder knowing a key Kab: either the key is a corrupted long-term key or a session key generated by the server for some dishonest agent. These two invariants suffice to derive the strengthening of the authorization guard (8).

The strengthening of the initiator authentication guard (10) corresponds to the following proof obligation. It describes the authentication guarantee that the responder B gets about the initiator A's state from receiving messages M2b and M3.

$$\begin{array}{l} \mathsf{Secure}(\mathsf{S}, B, [\mathit{Kab}, A, \mathit{Ts}]) \in s.\mathit{chan} \land \mathsf{dAuth}(\mathit{Kab}, [A, \mathit{Ta}]) \in s.\mathit{chan} \land \\ A \notin \mathit{bad} \land B \notin \mathit{bad} \rightarrow \\ \quad \exists \mathit{Ra}, \mathit{nl}.\ s.\mathit{runs}(\mathit{Ra}) = (\mathsf{Init}, A, B, \mathit{Kab}\#\mathit{Ts}\#\mathit{Ta}\#\mathit{nl}). \end{array} \tag{11}$$

When trying to prove that this is an invariant, we get stuck in a proof state that suggests replacing the honesty of A and B by the secrecy of Kab. This enables the successful completion of the proof.

$$\begin{array}{l} \mathit{M3}_{\mathit{krb2}} \equiv \{s \mid \forall \mathit{Kab}, A, B, \mathit{Ts}, \mathit{Ta}. \\ \quad \mathsf{Secure}(\mathsf{S}, B, [\mathit{Kab}, A, \mathit{Ts}]) \in s.\mathit{chan} \land \mathsf{dAuth}(\mathit{Kab}, [A, \mathit{Ta}]) \in s.\mathit{chan} \land \\ \quad \mathit{Kab} \notin \mathit{extr}_{\mathit{ik}_0}(s.\mathit{chan}) \rightarrow \\ \qquad \exists \mathit{Ra}, \mathit{nl}.\ s.\mathit{runs}(\mathit{Ra}) = (\mathsf{Init}, A, B, \mathit{Kab}\#\mathit{Ts}\#\mathit{Ta}\#\mathit{nl})\}. \end{array}$$

To establish the guard strengthening (11) using the invariants $\mathit{M2b}_{\mathit{krb2}}$ and $\mathit{M3}_{\mathit{krb2}}$, it suffices to show that the premises of (11) imply $\mathit{Kab} \notin \mathit{extr}_{\mathit{ik}_0}(s.\mathit{chan})$. This follows from an invariant stating that session keys with valid timestamps are never leaked and the following invariant, which guarantees to the server that the intruder never learns a session key generated for honest agents unless it has been leaked.

$$\begin{array}{l} \mathit{ikk\_srv}_{\mathit{krb2}} \equiv \{s \mid \forall \mathit{Rs}, A, B, \mathit{nl}. \\ \quad s.\mathit{runs}(\mathit{Rs}) = (\mathsf{Serv}, A, B, \mathit{nl}) \land A \notin \mathit{bad} \land B \notin \mathit{bad} \land \mathit{Rs}\$\mathsf{sk} \in \mathit{extr}_{\mathit{ik}_0}(s.\mathit{chan}) \rightarrow \\ \qquad (\mathit{Rs}\$\mathsf{sk}, A, B, \mathit{Na}, \mathit{Ts}) \in s.\mathit{leak}\}. \end{array}$$

This completes our sketch of the refinement proof of $\mathit{step5}_{\mathit{krb1}}$ by $\mathit{step5}_{\mathit{krb1}}$.